DOCUMENT RESUME

ED 127 132 SE 020 686

AUTHOR	Coburn, Terrence G.; And Others
TITLE	Michigan Educational Assessment Program, Mathematics Interpretive Report, 1973 Grade 4 and 7 Tests, Monograph No. 7.
INSTITUTION	Michigan Council of Teachers of Mathematics.
SPONS AGENCY	Michigan Education Association, East Lansing.
PUB DATE	Jan 75
NOTE	66p.; For monographs 5-9 of this series, see SE020684-688
AVAILABLE FROM	MCTM, Box 16124, Lansing, Michigan 48901 ($1.50 each, prepaid)
EDRS PRICE	MF-$0.83 Plus Postage. HC Not Available from EDRS.
DESCRIPTORS	*Achievement; Curriculum; *Educational Assessment; Elementary School Mathematics; Elementary Secondary Education; *Evaluation; *Instruction; *Mathematics Education; Secondary School Mathematics; State Programs; Testing Programs; Test Interpretation
IDENTIFIERS	*Michigan

ABSTRACT

In the fall of 1973, the Michigan Educational Assessment Program (MEAP) began using criterion referenced tests rather than the norm referenced tests used in earlier years. This volume presents the results of the 1973 study, presenting interpretive remarks about the performance levels of students from the fourth- and seventh-grades for each objective tested. Where appropriate, suggestions for curricular or instructional change are offered. Objectives are organized under 11 major topics: pre-number, numeration, addition, subtraction, multiplication, division, fractions, decimals, ratio, measurement, geometry, algebra, and graphing. Some objectives were tested at one grade level, others in both 4th and 7th grades. For each objective, this volume provides a sample item, remarks concerning the performance of the population tested, and suggested classroom activities related to the objective. In summary remarks, words and symbols which caused students difficulty are identified, and the overall results for each major topic are discussed. An appendix provides the score distributions for sets of items which test mathematical and reading objectives. (SD)

****
* Documents acquired by ERIC include many informal unpublished *
* materials not available from other sources. ERIC makes every effort *
* to obtain the best copy available. Nevertheless, items of marginal *
* reproducibility are often encountered and this affects the quality *
* of the microfiche and hardcopy reproductions ERIC makes available *
* via the ERIC Document Reproduction Service (EDRS). EDRS is not *
* responsible for the quality of the original document. Reproductions *
* supplied by EDRS are the best that can be made from the original. *
****

THE MICHIGAN

M
C
T
M

THIS DOCUMENT ENTERED LEVEL II

PER INSTRUCTIONS MCTM.

MICHIGAN EDUCATIONAL ASSESSMENT PROGRAM
MATHEMATICS INTERPRETIVE REPORT

1973 Grade 4 and 7 Tests

Terrence G. Coburn
Leah M. Beardsley
Joseph N. Payne

Monograph No. 7
January 1975

$150

Additional copies of this monograph can be obtained at [redacted] each (prepaid) by writing to.

Make checks payable to Michigan Council of Teachers of Mathematics (M.C.T.M.).

Other Monographs available from same address:

1. Classroom Proven Motivational Mathematics Games ($.60)
2. An Introduction to the Minimum Performance Objectives for Mathematics Education in Michigan ($1.00)
3. Order in Number ($1.00)
4. Metric Measurement Activity Cards ($2.00)
5. An Activity Approach to Fractional Concepts ($1.00).
6. Michigan Fourth and Seventh Grade Student Performance in Mathematics 1970-1973 ($1.50)

MICHIGAN EDUCATIONAL ASSESSMENT PROGRAM

MATHEMATICS INTERPRETIVE REPORT

1973 GRADE 4 AND 7 TESTS

Terrence G. Coburn, Chairman — Oakland Schools

Leah M. Beardsley — Detroit Public Schools

Joseph N. Payne — the University of Michigan

October 1974

ERIC

FOREWORD

Assessment is a new force bearing on the behavior of the teacher, along with old familiar forces such as textbook series, college courses, and supervisors. Whether assessment will be a potent force or a weak force remains to be seen. There is concern whether assessment will be a positive or negative force. If it proves to be a potent and positive force, it will not be due to the threat that is inherent in any evaluation of our work. Pressure to improve, cannot have a positive effect unless accompanied by direction and guidance.

The posture of many teachers regarding assessment has been defensive. They feel unjustly blamed for apparent deficiencies in their pupils. Teacher trainers, supervisors, and state officials are little threatened, feeling that they "do their best", and it is really up to the teacher to interpret, or implement, to bring the collective wisdom to bear on the child. It is our hope that all of these who shape classroom instruction will study assessment results and ask themselves whether their "piece of the action" might be changed in some way with an eye to improving the results.

This report suggests some areas in which guidance in needed. All those who play a significant role in shaping the teacher's ultimate performance should ponder the suggestion that children need to manipulate concrete materials and draw diagrams as they study arithmetic. Why, after at least 40 years of talk about the need for concrete activity do we find this suggestion necessary? Why is there such apparent

weakness regarding place value and grouping? Measurement is a relatively uncomplicated topic. Why is it showing up as a weak area?

It is not enough, nor is it fair, to ask these questions only of the classroom teacher. The teacher trainer should look at his role, the author at his elementary textbooks, the supervisor at his in-service training program.

Our familiar standardized normative tests gave us a badly blurred view of what children were learning. We have a somewhat clearer view through an assessment instrument based on specified objectives. It is hoped that this report will be but one document in a new sort of dialogue in mathematics education -- a dialogue based on what children can do, as measured against specific things we want them to be able to do.

William Swart
MCTM President

TABLE OF CONTENTS

I

INTRODUCTION

The Michigan Educational Assessment Program (MEAP) began in the fall of 1969. Norm-referenced tests in mathematics and reading were administered each January in the years 1970-1973 to nearly all public school fourth and seventh graders in Michigan. From 1971 through 1973, minimal performance objectives were developed and adopted by the State Board of Education in reading and mathematics.[1] Objective-referenced tests were given in the fall of 1973 to replace the previous testing. The results of the assessment can benefit those interested in mathematics education at various levels. All teachers of mathematics can benefit by utilizing local and state-wide results in their efforts to be more diagnostic in their instruction.

This monograph is concerned with the 1973 results for the mathematics objectives on the Grade 4 and Grade 7 tests. Its major purpose is to make interpretative remarks about the performance levels for each objective tested and to make relevant teaching and curriculum suggestions.

This monograph will be of use to those engaged in local interpretation or staff development. Local priorities and expectations should be considered along with this report in making an evaluation of a district's results.

---

[1] The reader is urged to secure a copy of Minimal Performance Objectives For Mathematics Education In Michigan, Michigan Department of Education, 1973. A helpful companion booklet is An Introduction To The Minimum Performance Objectives For Mathematics Education In Michigan, Monograph No. 2, February 1973, Michigan Council of Teachers of Mathematics.

II

PRESENTATION OF DATA

The writers of this monograph used several resources in arriving at their conclusions. These included the recomendations made at the MCTM/MEAP Conference, April 8-9, 1974 and a critical reading of their work by a reference group of leading mathematics educators in the state. Professional experience and judgement, statewide percent of attainment for each objective, and foil analysis data (Percent choosing each choice for each test item) were major ingredients in forming the interpretations.

In the fall of 1973 testing, 35 of the objectives classified as minimal were assessed in Grade 4 and 45 were assessed in Grade 7. Five multiple-choice test items were used for each objective. To be recorded as attaining an objective, a student must have responded correctly to 4 or 5 of the test items on that objective. The percents of students who attained each objective are summarized in Tables 1 and 2. On the left column are the content areas. At the top are ranges of percents of students attaining the objectives. The numbers inside the tables identify the objective. In Section III, each objective is described in full and the actual percent attaining the objective is reported.

By a minimal objective, the writers of this monograph mean an objective which nearly all Michigan students can and should master. Some of the reasons why a particular objective should be mastered are: (1) it contributes to developing literate citizens, (2) it is necessary for future employment, (3) and it is a prerequisite for subsequent learning in mathematics.

The actual percent of students acceptable as representing "nearly all" of the tested students is an elusive figure. The writers consider 85% as an acceptable level. One would like to think that 100% of the students should attain an objective if it is truly minimal. Testing conditions and other factors make such an expectation not reasonable.

TABLE 1

FOURTH GRADE TEST

PERCENT OF STUDENTS ATTAINING EACH OBJECTIVE

TOPIC	85-100	70-84	55-69	40-54	0-39
Pre-Number	1* 2 4 5	3			
Numeration	9 10 14	6 11 12	7 8 17[b]	21[c]	
	18	13 15 16	20		
		19[b]			
Addition	22				
Subtraction	23 25 27	28	24 26		
Fractions					29[c]
Measurement:					
Linear		30			
Time	31				
Money		32			
Temperature		33			
Geometry		34			
Algebra	35				

*These numbers are the objective identification numbers for testing purposes. See Section III for full description.

b and c refer to special designations described on page 5. Objective numbers without designating letters are judged to be in category a.

TABLE 2

SEVENTH GRADE TEST

PERCENT OF STUDENTS ATTAINING EACH OBJECTIVE

TOPIC	85-100	70-84	55-69	40-54	0-39
Numeration	2	1			
Addition	3 4				
Subtraction		5			
Multiplication	7 8	6 9			
Division	10	11[b] 12	13		
Fractions:					
Meaning		15	14 16		
Addition			17 18		
Subtraction			19	20	
Multiplication		21			
Decimals:					
Meaning		22			23
Add/Subt		24			25 26
Ratio					27[b,c]
Measurement:					
Linear				28	
Area				29	
Volume					30[b]
Time			31 32		
Money		33	34	35	
Temperature		36			
Geometry		39	37	38	
Algebra	42	40 44	41	43[c]	
Graphing					45[c]

The interpretation categories used by the writers were modified from those given by Womer in a memo on the assessment results.[1] The categories and their designation in Tables 1 and 2 are as follows:

a. Minimal and appropriate. The objective is judged to be minimal and the test items appear to be appropriate. If the level of attainment is not at or above 85%, it may be due to inappropriate instructional materials or lack of proper emphasis in years prior to the test year. Objectives in this category are unlabeled in the tables.
b. Poor test items. The objective is judged to be minimal but the item(s) have test construction flaws which probably contributed to the low level of attainment. Also, an item may not have been a valid measure of an objective.
c. Inappropriate. The writers feel that the objective, as currently written, is not a minimal objective.

III

RESULTS, REMARKS AND SUGGESTIONS

The results of the Grade 4 and Grade 7 test are reported in the following pages. The objectives are grouped in sections according to content. A three-column format allows results, remarks, and teaching suggestions to be viewed as conveniently as possible. The statewide result and the objective description are located in the lefthand column. The remarks concerning the level of performance are in the middle column and are aligned with each objective. Teaching and curriculum suggestions are in the righthand column. These suggestions are made for the entire content section and are not aligned with each objective.

[1]Womer, Frank B., "Half A Loaf-An Evaluation of the 1973 MEAP Results", A mimeographed paper, the University of Michigan Bureau of School Services, School of Education, 401 S. Fourth Street, Ann Arbor, Michigan 48103.

EXPLANATION OF MATERIAL GIVEN IN THE OBJECTIVE AND RESULTS COLUMN

a b c d

(a) 1. (b) AR-I-A-6 (c) (Grade 4) (d) 85%

(e) Indicate Objects That Are Same Size

(f) Items 96-100

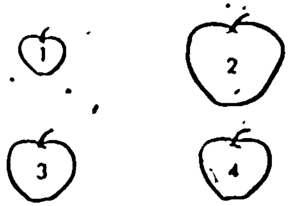
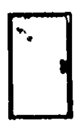
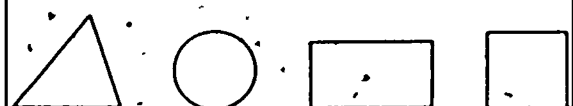
(g) Given a set of objects, the learner will recognize objects that are the same size.

- a. The objective identity number assigned for this test.
- b. The objectives full code number as it appears in MPOMEM.
- c. The objective is to be mastered by the end of Grade 3. and was tested at the beginning of Grade 4.
- d. Eighty-five percent of all students tested got either 4 or 5 out of 5 test items correct for this objective. See APPENDIX A to see the percents for zero correct, 1 correct, 2 correct, 3 correct, 4 correct, and 5 correct.
- e. The short title for the objective as given on the MEAP data reports.
- f. The item numbering for the five test items.
- g. The description of the objective being tested.[2]

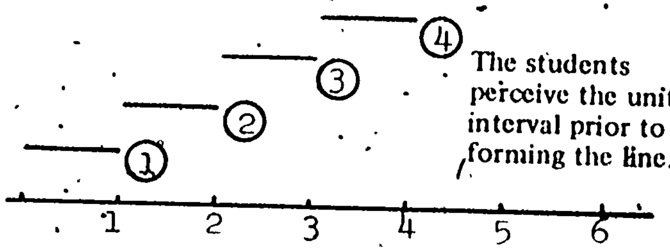
---

[2] Several of the minimal performance objectives were rewritten by the MEAP staff into a form more appropriate for paper and pencil testing in a group administered multiple choice format. This revised version is given in Section III. The reader should review the wording of the original objectives in <u>Minimal Performance Objectives For Mathematics Education In Michigan</u> (MPOMEM).

The writers recognize the difficulties inherent in group administered multiple choice tests. We hope that teachers will continue to use concrete objects for both instruction and individual assessment. We further urge that MEAP not alter any objectives from the concrete to the picture-identification format when such an alteration changes the nature of the skill or conept involved.

OBJECTIVE	RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
1. AR-I-A-6 (Grade 4) Indicate Objects That are Same Size Items 96-100 Given a set of objects, the learner will recognize Objects that are the same size. SAMPLE: Which of these apples are the same size? 	85%	1. Student performance was quite good on same size.	1. Check the vocabulary of same size and shape using real objects. 2. To stress size difference, use nested boxes or wooden dolls and ask: Which is the largest? Where will you find the smallest? Why? Are any of the dolls the same size? 3. To reinforce the concept of similar geometric shapes: (a) Stress that many objects have the basic shapes of either a triangle, a rectangle, a square, or a circle. (b) Use. (1) Geo-Strips (strips of cardboard with holes punched at each end). Fasten these strips together with paper fasteners. Have children make a shape like a tree; like a window; like a bed. (2) Pictures: What objects in our room have the same shape as this picture of the moon? (3) I-Spy: Have children spy shapes as they play this game. "I spy an object that is about the shape of a square, who can guess it?" (4) Think — describe the doors in your house.
2. AR-I-A-8 (Grade 4) Indicate Similar Geometric Shapes Items 91-95 Given an object shaped like a circle, triangle, square, or rectangle, the learner will choose the shape the object represents. SAMPLE: Which figure below has about the same shape as this door?  	90%	1. Student performance was quite good on same shape. 2. Some students may have had difficulty with the vocabulary of "same size" or "same shape". Others may need strengthening of their visual discrimination. The reader is directed to the summary where some general observations are made about vocabulary.	

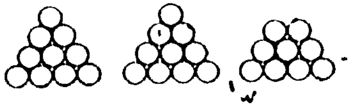
OBJECTIVE	RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
3. AR–I–A–16 (Grade 4) Indicate Objects Arranged Full to Empty Items 116–120 Given a set of three containers, one full, one empty, and one half-filled, the learner will choose the containers that are arranged from full to empty. SAMPLE: Given four pictures, identify the picture that shows three containers in a full to empty order "Which picture shows the glasses in order from full to empty?"	71%	1. Twenty-nine percent of the students did not achieve this objective. Nine percent answered none of the five items correctly. Incorrect choices were equally divided among the distractors. 2. The main difficulty appears to be with the vocabulary of "order from full to empty". There is evidence here and in other results in this assessment that students have difficulty with the technical vocabulary of mathematics in general and specifically with order. 3. The object is minimal.	1. There is a need for more experience with arranging concrete objects in order. The essential vocabulary of full, middle, empty, last, first, etc. must be taught in conjunction with concrete aids. Care should be taken in teaching antonyms like full/empty, first/last. Students often are able to perceive first from last but they cannot connect the correct term with their perception. 2. Teach full to empty by using plastic glasses and sand. Have the students fill a glass, partially empty the glass, and empty the glass. Say, at the appropriate times, "Now my glass is empty." Then use three separate glasses full, part full, empty, work on identifying and arranging in order from first to last and from full to empty.
4. AR–I–A–24 (Grade 4) Indicate Longest and Shortest Objects Items 106–110 Given a collection of five objects of varying lengths, the learner will identify the longest or shortest as requested. SAMPLE: Identify the longest or shortest from four pictures of an object. "Which pencil is the shortest?"	90%	1. The level of performance is not outstanding, but it is adequate. The error most often made involved choosing the opposite, e.g., shortest when longest was correct and vice versa.	3. Interpret the meaning of the suffix "est" on longest and shortest. Have the student tell in his own words that he can find no longer nor shorter one in the set. 4. Review the rules for judging which is first When objects or pictures in a line have a front (car) or a face (cat), they are ordered by the way they are facing. When objects do not have a front or face (box), they are ordered from left to right.
5. AR–I–A–39 (Grade 4) Indicate First and Last Items 101–105 Given five small toys in a line, the learner will identify the first toy and the last one. SAMPLE: Given 5 pictures of an object in line, identify the first or the last. "Which turtle is last?"	87%	1. The percent correct per item ranged from 87 to 92. Those who made an error usually picked the opposite, e.g., last for first and vice versa. The percent selecting the opposite end of the line ranged from 2 to 6 percent.	5. Use small toys and have a child arrange them in line with the red car first, and let him select the toy that he wishes to place last. Change the order so the red car is last and his favorite toy is first. 6. Think: When do you want to be first in a line? When do you want to be last?

OBJECTIVE / RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
6. AR–I–B–7 (Grade 4) 84% Choose Equivalent Sets Items 126–130 Given a set with less than ten objects, the learner will identify an equivalent set. SAMPLE: Identify a set with the same number as a given set. "Which group below has the same number of members as this group?" 	1. There is no doubt that nearly all students can attain this objective by grade 4. While the performance was satisfactory, the language and general layout of the test items were not familiar to most students and the results are probably artificially low.	1. Have students match concrete objects in a 1-to-1 correspondence. Children to desks–"same number", books to children–"same number". 2. Make sets that have the same number as a given set. Show a set of dogs and have the students draw dog houses so they have the same number of of houses as dogs. 3. Connect the word fewer with the word less. Ask, "How can you make a set having fewer numbers when it is a set of children?" "a set of candy?" "etc."
7. AR–I–B–32 (Grade 4) 67% Choose Sets Having Fewer Numbers Items 131–135 Given a set of two to eight objects, the learner will identify a set having fewer members than the original set. SAMPLE: "Which group below has fewer members than this group?" 	1. Twenty-six percent missed four or five items. Half missed at least one item. The range of percent correct per item was from 65 to 77. 2. In four of the five items, a picture showing the same number of objects was available as an incorrect response. The range of percent selecting this incorrect response was from 14 to 25. This could be due to the influence of the previous set of five items where the students were directed to to identify the group having the same number of members (Objective 6). The overall form of the items is similar for Objectives 6 and 7. 3. Fewer may be a more difficult word for students than the word less. The objective is minimal, however, some consideration should be given to revising the objective to use the word less rather than fewer.	4. Perception cards. A child draws and shows a card to the class. Another child draws and compares his card to the first child's; if it has more objects he must sit, but if it has fewer he remains standing and the first child sits. Continue until all the cards are drawn and the last child standing is holding the card with the "fewest" objects.
8. AR–I–B–40 (Grade 4) 68% Indicate Appropriate Numeral For Point On A Line Items 161–165 Given a line marked with congruent segments and a set of number cards (0-10), the learner will choose the appropriate number card for the point on the line. (continue to page 10)	1. Approximately 11 to 18 percent per item selected that response which indicates counting the points instead of the intervals on the number line. 2. The objective is minimal.	5. Use straws in building a model of the number line. The students learn to count the intervals instead of the dots. Individual cards with the numerals 1, 2, 3, . . . can be taped at right hand end of each straw for order. As you develop the number line from a stairstep to the line, zero should come in last.  The students perceive the unit interval prior to forming the line.

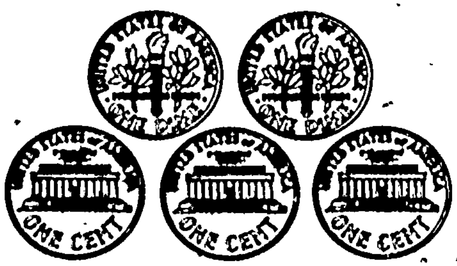
OBJECTIVE RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
SAMPLE: Choose a whole number for a point on a number line, marked by an arrow. The first two points on the number line are labeled 0 and 1 respectively.		
	NUMERATION-ORDERING	
9. AR–I–B–43 (Grade 4) 86% Choose Greatest and Least Number Items 36–40 Given any three numbers, 0-10 the learner will identify which number is the greatest and which is the least, on request. SAMPLE: Which number is least? A. 4 B. 3 C. 6	1. The most common error involved identifying the opposite, e.g., the greatest number of the three given numbers was identified as the least. The range of percents selecting this incorrect answer was 6-8. This result is in keeping with the results regarding objectives involving opposites.	1. Have the students recognize that the greatest means most. Use follow-the-dot pictures to the greatest number. Jump a cardboard frog from the least to the greatest number seen on the number line. Shown sets of three numbered cards, the child selects the greatest in each set.
10. AR–I–B–44 (Grade 4) 88% Choose Number Between Two Numbers Items 71–75 Given two consecutive even or odd numbers, 0-9, the learner will name the number that comes between the two given numbers. SAMPLE: Which number comes between 5 and 7?	1. Eighty-four percent answered all five items correctly. A common error was to select the number which comes just before the lesser of the two given numbers.	2. Show-Me-Cards: Students fill in individual 3-pocket cards as the teacher says, "Put a 7 in the first pocket. Put a nine in the last pocket. Put the number that is between 7 and 9 in the middle pocket." Ask students to place a 7 in the middle pocket. Next place the number that comes before 7 in the first pocket – and the number that comes after 7 in the last pocket.
11. AR–I–B–45 (Grade 4) 80% Choose Number Before or After a Number Items 56–60 Given a number from 1 to 8, the learner will identify the number that comes before or after the given number. SAMPLE: Which number comes just before 5?	1. The range of the percent correct per item was 80-89. 2. As with the other items involving opposites, some of the students were prone to identify the number just before the given number where the correct answer was the number just after. Sixteen percent chose 7 on Item 58, "Which number comes right after 8." 3. The objective is minimal. The difficulty seems to again be with the specific vocabulary.	Have individual students write consecutive numbers on five separate cards, e.g., 46, 47, 48, 49, 50. Each student holds his cards in a deck and moves them from front to back to find and show the card that answers each question. "What number comes before 49? Show me. After 46? Show me."

OBJECTIVE	RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
13. AR–I–B–65 (Grade 4) Indicate Number Before or After Number Within A Decade Items 66–70 Given a set of sequentially ordered whole numbers within a decade less than 100, such as 31, 32, . . . 40, the learner will identify the number that comes immediately before or after a given number, as requested. SAMPLE: Which number comes just before 22 in the series (20, 21, 22, 23, 24)?	71%	1. One half of the students missed at least one item. The range of percent correct per item was from 77 to 83. 2. Again, a common error was to select the opposite choice to that required. The objective is minimal and these results further indicate the need to give more attention to specific language instruction in mathematics.	
14. AR–I–B–67 (Grade 4) Indicate Which Of 2 Numbers Is Greater or Less Items 41–45 Given 2 two-digit numbers, the learner will tell which number is greater and which number is less. SAMPLE: Which number is greater? F. 29 G. 31	89%	1. The level of performance is good. While one would expect about 50 % of the students to attain the objective by guessing, the results are adequate enough to interpret them as indicating that the students are familiar with "greater" and "less".	

ERIC Full Text Provided by ERIC

OBJECTIVE	RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
12 AR I B 64 (Grade 4) Identify A Numeral Less Than A 100 Items 121 125 Given a set of tens and ones representing a number less than 100, the learner will identify the numeral. SAMPLE: How many balls are in this group? Answer 29 	78%	1. The items match the description of the test objective, however, the wording has been altered from the MPOMEM version "... learner will say and write the numeral." 2. There were no apparent patterns in the selection of wrong answers. 3. The results must be interpreted cautiously because many children may have counted by one instead of using the grouping by ten. 4. The objective is minimal	1. Successful instruction with the computational algorithms is dependent upon mastery of expanded notation and other place value and base concepts. Textbooks are largely at fault for depicting expanded notation in ways that children can solve exercises through the use of patterns not really understanding the base and place value concepts. More work with base is needed in the first grade, especially work with concrete objects like bundles of sticks. 2. Using buttons or other counters, the child counts by ones until he has collected a set of 10 ones. He then wraps up his set of ten by drawing a line around it, and labels it "1 ten". The child builds teen numbers with his ten set and extra ones and records these numbers. The child builds enough ten sets and extra ones to represent the number 23. The child uses bundles of ten sticks wrapped with rubber bands and extra one sticks. Ask him to show 34. His arrangement should have the 3 tens on the left and the 4 ones on the right, or 34. Practice many numbers to 99. The child strings beads on telephone wire to make sets of ten, and sets of ones. These are hung to illustrate any number from 1 to 100.
16. AR I-B 79 (Grade 4) Identify Correct Expanded Notation Items 81-85 Given any three-digit number, the learner will write expanded notation to 1,000, first by using place value words and then by using numerals. SAMPLE: Which one of these is 495? Answer: 400 + 90 + 5	74%	1. It is felt that only Item 83 correctly measured the objective. The distractors in the other items did not match the order of the digits in the given number. For Item 83, 44% correctly identified 600 + 50 + 9 for 659 and 30% selected 6 + 5 + 9. Nineteen percent chose 6 + 50 + 9 and 6% chose 60 + 5 + 9. The results for this objective are probably artificially high. 2. The objective is minimal and the indication is that numeration concepts need more adequate instruction in grades 1-3.	

NUMERATION–MONEY

OBJECTIVE / RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
15. ~~AR-I~~-B–70 (Grade 4) 78% Indicate The Values Of A Set of Dimes and Pennies Items 171–175 Given a set of dimes and pennies valued between 11 and 99 cents (one dime, one penny to nine dimes, nine pennies), the learner will state the value. SAMPLE: How much are these coins worth? 	1. Three items used pictures of coins such as shown in the sample. The form of the other two items was, "How much are 5 dimes and 6 pennies worth?" There was no apparent difference in performance between the items with coins and the items without coins. The percent correct on the former were 76-82 and 82-85on the latter. 2. Eleven percent selected 50 ¢ as their answer to the sample. One can see where these students could have counted five dimes, not recognizing the pennies from the dimes. 3. The objective is minimal. A little more attention given to money will increase these results.	1. Children are asked to select certain coins from a pile of actual money, play money, and pictures of money. Both sides of coins should be tested during these activities.

NUMERATION-SEQUENCE

OBJECTIVE / RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
17. AR–I–B–81 (Grade 4) 64% Choose List of Numbers In Ascending Order Items 61–65 Given a random list of two- and three-digit numbers, the learner will identify the list that is in ascending order. SAMPLE: Identify the group of five numbers that are shown from least to greatest.	1. This was one of the five lowest performances on the Grade 4 test. Eight percent failed to answer any item correctly and 27% missed at least 3 items. 2. The low performance on this objective is related to the low performance on vocabulary items involving opposites, "least/greatest" and also to the overall low performance on items involving ordering or sequencing. 3. Perhaps four sets, each with five 2-and3- digit numbers, constituted too many numbers for "end of grade three" students to handle. 4. The objective should be revised to require that the ordering include tens only as well as tens and hundreds mixed. The sets should also be limited to 3 or 4 numbers. The test items could be improved by drawing a box around each set.	1. Introduce the concept least to greatest by having children consider and arrange only three numbers - 78 49 100. When the concept has been fully understood and this skill has been attained, introduce four numbers and when children are successful with these, attempt five. 2. To learn the art of comparing the quantitative size of two numbers, use concrete objects to illustrate two digits 7 and 9. Progress to 7 and 17; to 13 and 31; 34 and 43; 64 and 46; 81 and 100; etc. Illustrative material should point out that any 2-digit number is less than any 3-digit number. 31 = $\boxed{10}$ $\boxed{10}$ $\boxed{10}$ $\boxed{1}$ 131 = $\boxed{100}$ $\boxed{10}$ $\boxed{10}$ $\boxed{10}$ $\boxed{1}$

OBJECTIVE — RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
18. AR-I-B-82 (Grade 4) 86% Indicate Greater or Less/Scrambled Positions Items 46–50 Given 2 three-digit numbers which have the same digits but in different positions, the learner will compare them to determine which is greater and which is less. SAMPLE: Which number is least? F. 297 G. 792	1. The results are satisfactory and similar to that for objective 14, AR-I-B67.	Use this same technique to compare two 3-digit numbers. 431 = 100 100 100 100 10 10 10 1 134 = 100 10 10 10 1 1 1 1
19. AR-I-B-84 (Grade 4) 77% Indicate Next Number In A Sequence Items 51–55 Given a counting sequence of two to four nembers, the learner will write the next number in the sequence. SAMPLE: Which number comes next? 2, 4, 6, answer: 8	1. The question, "Which comes next?" is ambiguous. Thirteen percent felt that 7 came next as an answer to the sample. 2. It is desirable, and minimal, that students at the end of grade 3 can count by twos, fives, and tens. The pattern/sequence type of skill as shown by these test items is not minimal.	3. Show many cards with pairs of numbers written one above the other. Children are to judge whether or not they are able to subtract the numbers as they are arranged and state the reason why or why not. 749 − 947 82 − 79 100 − 94 126 − 621

NUMERATION-NUMBER MEANING-Multiples of Two

OBJECTIVE — RESULTS

20. AR–I–B–85 (Grade 4) 68%
Indicate a Number That Is A Multiple of 2
Items 76–80

Given the counting numbers 1-10, the learner will indicate those that are multiples of 2.

SAMPLE: Which is an even number?

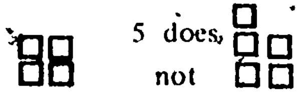
A. 1 B. 2 C. 5 D. 7

REMARKS

1. The objective does not call for the recognition of the term "even". It is felt, however, that by grade 3, nearly all students should know that an even number is a multiple of 2.
2. Nineteen percent missed at least four items.
3. The major difficulty appears to be with the vocabulary, i.e. "even number". The objective is minimal.

TEACHING AND CURRICULUM SUGGESTIONS

1. The terms even and odd should be understood and recognized by the end of grade 3. Introduce the concept with objects to count and use the word level. Stack the objects in two columns.



4 makes a level stack — 5 does not

Connect even to level–comes out even.
Emphasize finding even numbers first. Then later introduce the word odd.

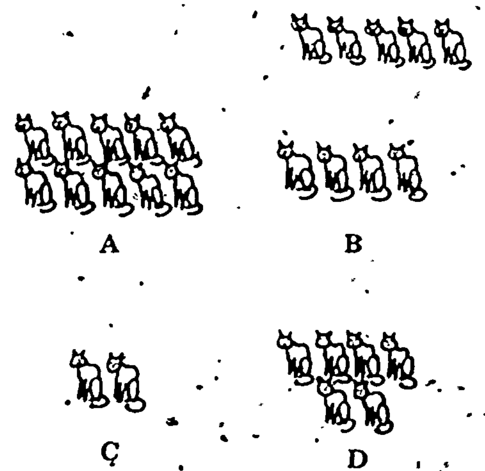
Hmm! that's good

OBJECTIVE — RESULTS

21. AR–I–B–86 (Grade 4) 47%
Select Set With Twice As Many Members As Another
Items 136–140

Given a set of objects, the learner will select another set that will have twice as many objects.

SAMPLE: Which group below has twice as many cats as this group?



REMARKS

1. A very poor performance. Clearly the terminology twice as many is unfamiliar to the majority of the students at the end of grade 3.
2. The range of percent correct per item was 52-69. Many of the students appear to have translated "twice as many" to mean "two more than". The range of percents selecting the type of incorrect response was 14-23.
3. As given, the objective should be regarded as minimal with some change in wording. The concept, "twice as many" is attainable by a majority of the K-3 students. The language of the test items, however, is not familiar to the student. We recommend that the objective and the test items be revised to use the language: two times as many. For example, Which group below has two times as many cats as this group?

TEACHING AND CURRICULUM SUGGESTIONS

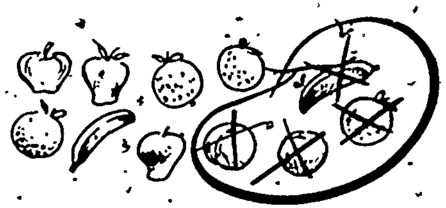
2. The meaning of twice as many is established as two times as many. If you have five fingers on one hand you have twice as many on two hands.

Have two boxes. Put 3 blocks in one box. Have child put 3 blocks in the other box so that now there are twice as many. The child says that 6 is twice as many as three. Use the two hands of the child for the same activity.

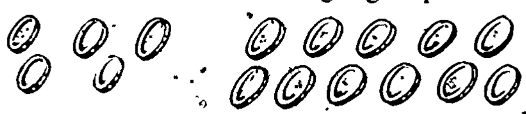
OBJECTIVE / RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
1. AR–I–B–87 (Grade 7) 76% Identify Number 100/1000 Larger Items 6–10 Given any four-digit number, the learner will identify the number that is 100 or 1000 more or less than it is, without using formal addition or subtraction. SAMPLE: What number is 100 more than 5,897? F. 5,997 G. 5,097 H. 5,998 J. 6,897	1. On the sample, 26% selected choice J which is 1000 more. On Item 9, given the same directions as in the sample, 17% chose the response which 1000 more. 2. Students performed better when directed to find the number that was 100 (or 1000) less than the given number. The results was 90% correct for each item versus 67 to 79 percent for the items requiring 100 (or 1000) more. 3. Better distractors may have caused a lower performance. For example, neither 6,346 nor 5,246 were choices for Item 8, "What number is 1,000 less than 5,346?". 4. The objective is minimal. The results add to the mounting evidence that numeration is not receiving adequate attention in the elementary mathematics program.	1. Teaching to read and write numbers is important and the results for objective 2 are encouraging. 2. More efforts need to be directed to removing the difficulties as seen in objective 1. Use place value strips placed above the chalkboard, so that a number written below the strips has each digit under its proper place name. Under this numeral write 100, and add (or subtract) to obtain the answer. This procedure should lead to the mental short-cut of adjusting only the digit in the hundred's place. Similarly for adding or subtracting 1000. What number is 100 more than 5,897? T H Tens Ones 5 8 9 7 + 1 0 0 5 9 9 7 a) Write the 100 in the proper place. b) Does 100 more mean add or subtract in the hundreds place? c) Write and read the answer.
2. AR–I–B–89 (Grade 7) 95% Identify Arabic Numeral Items 1–5 Given a number orally, the learner will identify the Arabic numeral. SAMPLE: Identify the 3- or 4-digit numeral for the number read aloud by the teacher.	1. Results are very good. This objective had the greatest percent of attainment for any objective on either test. 2. It should be noted that this objective is related to the connection between oral and written expression for numbers. This aspect of numeration, which is useful and important, is not related to work with algorithms.	

ADDITION OF WHOLE NUMBERS

OBJECTIVE / RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
22. AR–II–A–10 (Grade 4) 93% Add Two-Digit and One-Digit Number/No Carrying Items 21–25 Given addition exercises involving a two-digit number plus a one-digit number requiring no regrouping (carrying), the learner will find the sums with or without the use of aids. SAMPLE: 43 +5	1. Eighty percent got all five items correct.	Performance on addition seems high overall. Keep up the good work. Perhaps time spent in total group instruction with addition in grades 5 and 6 can be reduced. Individual work can be done with individual difficulties and more time can be spent with some of the more troublesome topics.
3. AR–II–A–22 (Grade 7) 94% Add 3- and 1-, 2-, and 3-Digit Numbers Items 11–15 Given addition exercises involving a three-digit number plus a one-, two-, or three-digit addend, with or without regrouping (carrying), the learner will identify the sums, using any technique. SAMPLE: 857 + 36 = $\square$	1. Very good performance. 2. Two items were in horizontal form. The results slightly favored the vertical form.	
4. AR–II–A–26 (Grade 7) 88% Add 2 or 3 Numbers Items 16–20 Given addition problems involving two or three addends with three, four, five, or six digits, with or without regrouping, the learner will find the sums, using any techniques. SAMPLE: 2,371 50,452 + 938	1. For four of the items the performance ranged from 90 to 95 percent correct. On the item shown in the sample the percent correct was 77. About 16% did not rename (carry) from the hundreds to the thousands place correctly (they answered 52,761). 2. Only 66% got all five items correct.	

OBJECTIVE RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
23. AR–II–B–6 (Grade 4) 86% Identify Operation and Sum or Difference/No Borrowing Items 1–5 Given a word problem read by the teacher, involving addition or subtraction combinations to 10, the learner will identify the operation and the sum or difference. SAMPLE: Mary has 10 pennies. Linda has 6 pennies. Which number sentence tells how many more pennies Mary has than Linda? Answer: 10 ¢ – 6 ¢ = 4 ¢	1. One problem required addition and was done correctly by 95% of the students. Three problems required subtraction for "How many are left?" situations and the percent correct for each was 90 or above. 2. The sample problem shown at the left requires subtraction for the "How many more are needed?" situation. Only 65% got this problem correct with 25% incorrectly choosing the addition equation, 10 ¢ + 6 ¢ = 16 ¢. This poor result probably reflects the excessive emphasis on set removal for subtraction and inadequate attention to relating subtraction to this kind of problem.	There is no doubt that subtraction is more difficult than either addition or multiplication. Data from this test and from national tests support this observation. Still, however, it is not clear what the remedies should be. Addition is more practiced than subtraction because each check of subtraction requires addition. And there are three major physical situations which are solved by subtraction and only one to be solved with addition. Furthermore, the ways we teach children to find the answers to subtraction are not as well stated nor as clear as they are for addition. With these general observations in mind, these suggestions are offered: 1. Students need more help in representing the concept of subtracting using diagrams. Care is needed in identifying the "whole set" and the "part" which is to be subtracted. 2. Errors in the algorithm itself seem rooted in inadequate conceptual work with place value and with subtraction itself. Renaming errors are related to place value and reversals by failure to understand that subtraction is not commutative. a. Concrete objects or effective diagrams are needed to help with renaming, e.g.  2 tens 3 ones  1 ten 13 ones Then, this is related carefully with the written form.
24. AR–II–B–9 (Grade 4) 63% Number Sentences/Subtraction Items 146–150 Given a set of objects or pictures showing a subtraction relationship with combinations to 18, the learner will identify an appropriate number sentence. SAMPLE: Which number sentence below tells about this picture? Answer: 11 – 4 = 7 	1. The results on this objective are not satisfactory. Over one-third of the responses each test item was incorrect. A disturbingly high 17% got none of the five items correct. 2. The most common incorrect response was to name the number of members in the two subsets and then subtract, e.g., 7 - 4 = 3 for the sample item. This type of error accounted for over one-fourth of the incorrect responses for for each item. 3. The objective is minimal. More emphasis needs to be placed on constructing and interpreting diagrams. A local district may wish to retest this objective using its preferred diagram for subtraction situations.	

 (continued from page 18)

OBJECTIVE — RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
25. AR–II–B–11 (Grade 4) 85% Number Sentences/Addition or Subtraction—Identify Operation Items 6–10 Given a subtraction word problem read by the teacher involving combinations to 18, the learner will: (1) identify the operation, (2) identify an appropriate number sentence, and (3) identify the answer. SAMPLE: Joe has 14 ¢. Bob has 9 ¢. Which number sentence tells how many more cents Joe has than Bob? Answer: 14 ¢ 9 ¢ = 5 ¢	1. While the overall performance seems satisfactory, the results varied greatly depending on the kind of physical setting and the way the question was asked. The four problems using "Comparison" or "How many left?" situations seemed much easier (about 90 percent correct on each) than the one problem shown in the sample which asked "How many more?". On the sample problem 72 percent were correct with 20 percent incorrectly choosing the addition equation 14 ¢ + 9 ¢ = 23 ¢. 2. The results on this set of orally-stated problems is substantially better than on the test items using diagrams in the previous objective.	b. In teaching subtraction, make sure children discriminate $\begin{array}{r}7\\ \underline{3}\end{array}$ from $\begin{array}{r}3\\ \underline{7}\end{array}$ and are able to choose which one shows 3 subtracted from 7. This needs review when the algorithm is taught as well. c. Children need much more help in relating the language of "more" in "How many more are needed?" situations to the operation of subtraction.
26. AR–II–B–13 (Grade 4) 64% Numerical Set Comparisons Items 141–145 Given two sets of objects, one with more objects than the other, the learner will identify how many more members it has. SAMPLE: How many more buttons are in the larger group? 	1. Results on this objective are comparable to those using similar diagrams on objective 24. 2. A common incorrect response was to name the number of members in the set with more members. On the first item, the item shown as a sample, 56 percent got it correct and 25 percent chose the number 11 as the answer. 3. The objective is minimal. The "How many more?" situations are not receiving adequate initial work. "Take away" is overemphasized to the detriment of the other subtraction situations.	3. The relatively modest performance on 2-digit subtraction by the end of Grade 3 suggests that subtraction with renaming may be introduced too early in some schools (some do it in grade 2). Clearly, subtraction with renaming should not be done until subtraction without renaming is learned well.
27. AR–II–B–15 (Grade 4) 87% Subtract One-Digit from Two-Digit Number/No Borrowing Items 26–30 Given a two-digit number, the learner will subtract one-digit numbers with no regrouping (borrowing) with or without the use of aids. SAMPLE: $\begin{array}{r}69\\ \underline{-\ 8}\end{array}$	1. The percent correct on each item was about 90, with the highest of 92 for 26 and the $\underline{-\ 0}$ lowest of 86 on 69. The error patterns were $\underline{-\ 8}$ difficult to discern.	4. To illustrate subtraction with regrouping, tape clear plastic glasses to the chalkboard, and write the problem directly below them. The child places the correct number of sticks in the glasses. He must move one set of 10 to the ones glass in order to subtract in the ones place. This action is recorded and the problem finished. Place sticks — Regroup — Subtract  $\begin{array}{rr}3 & 2\\ \underline{-1} & \underline{8}\end{array}$ $\begin{array}{rr}\overset{2}{\cancel{3}} & {}^{1}2\\ \underline{-1} & \underline{8}\end{array}$ $\begin{array}{rr}\overset{2}{\cancel{3}} & \overset{12}{\cancel{2}}\\ \underline{-1} & \underline{8}\\ 1 & 4\end{array}$

 (continued from page 19)

OBJECTIVE / RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
28. AR–II–B–16 (Grade 4) 81% Subtract Two-Digit from Two-Digit Number/No Borrowing Items 31–35 Given a two-digit number, the learner will subtract a two-digit number with no regrouping (borrowing). SAMPLE: 98 −73	1. The range in percent correct on these five test items was 76 87. Twenty percent missed tow or more items. 2. The objective is minimal.	
5. AR–II–B–20 (Grade 7) 78% Subtract 2- or 3- Digit Number From 3- Digit Number Items 21–25 Given a three-digit number, the learner will subtract a two- or three-digit number, with or without the use of aids. SAMPLE: 625 −58	1. There was a range of 86 percent correct on 834 −279 to 74 percent correct on 800 −277. Other percents correct were in the low 80's. Twenty-two percent missed two or more items and 43 percent missed at least one item. 2. Two major sources of error seem apparent from a study of distractors–renaming and reversals. Renaming (borrowing) errors occurred when a number of hundreds or tens was not reduced when no renaming was required, e.g., getting 533 as the answer for 800 − 277. Reversals occurred when a smaller digit was subtracted from a larger digit, irrespective of the number containing the digits, e.g., solving 914 − 875 and getting 161. 3. The objective is minimal.	

MULTIPLICATION OF WHOLE NUMBERS

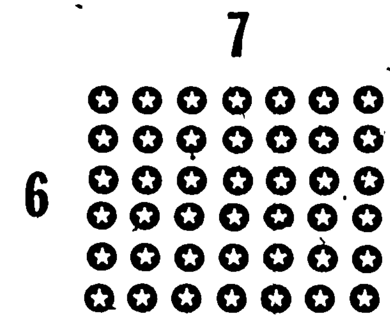
OBJECTIVE / RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
6. AR–II–C–6 (Grade 7) 83% Represent Repeated Addition As Multiplication Items 36–40 Given a repeated addition sentence, the learner will represent it as a multiplication sentence with its product. SAMPLE: 6 + 6 + 6 + 6 + 6 = 30 – means the same as which of the following sentences? Answer: 5 x 6 = 30	1. There were two major distractors: a) sentences using the correct numbers but + instead of x, e.g., 5 + 6 = 30; b) obviously true, sentences which do not use the correct multiplication sentence, e.g., 26 + 4 = 30 or 36 x 1 = 36. 2. The objective is minimal.	1. Ideas about multiplication same to have been learned well although some children need help with the language and symbolism for expressing the ideas. 2. Understanding the algorithm for multiplication depends heavily on place value. The tendency for errors to occur in place value suggests that explicit connections be made to place value when teaching any multiplication algorithm. e.g. . 3 x 4 = 12 so 3 x 4 tens = 12 tens 3 x 40 = 120 3 x 4 hundreds = 12 hundreds 3 x 400 = 1200
7. AR–II–C–11 (Grade 7) 93% Inverse Multiplication (Commutative) Items 41–45 Given two numbers, the learner will demonstrate that the order in which they are multiplied does not change the product. SAMPLE: Which means the same as 16 x 22? Answer: 22 x 16	1. Most students seem to understand commutativity by the end of grade 6. Performance was high. Most errors occurred by choosing responses using the same numbers but other operation signs. 2. The language "means the same as" reflected a different usage than in the items measuring objective 6. The writers feel that this item is basically not a good test of the commutative property.	3. Multiply 600 in vertical form, then use the x 3 1800 equation form, 3 x 600 = 1800. Verify the multiplication by repeated addition. 600 600 + 600 1800 4. Using the shortcut form of multiplying powers of ten by annexing zeros, utilizes the intrinsic nature of the decimal number system, of our currency system, and of the metric system.

ERIC Full Text Provided by ERIC

OBJECTIVE — RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
8. AR–II–C 13 (Grade 7) [illegible]8% Multiply 1-Digit Number and Multiple of 10/100 Items 56–60 Given a one-digit number (10,20. . .), (100, 200. . .), the learner will state the product. (Multiply 1-digit by multiples of 10 or 100) SAMPLE: $3 \times 200 = \square$	1. While the percent attaining criterion is high, it must be noted that 18 percent missed the problem $8 \times 400 = \square$. The major error was leaving off a zero. In another example in multiplying hundreds, an extra zero was appended.	5. Introduce multiplication beyond the multiplication facts by applying the distributive law. Place an array against the multiplication chart that goes beyond the chart, e.g., a 3 x 14 array. Fold the array in parts that fit the chart 3 x 10 and 3 x 4. Write: $3 \times 14 = (3 \times 10) + (3 \times 4) = 30 + 12 = 42$ Shorten to $\begin{array}{r} 14 \\ \underline{\times\ 3} \\ 12 \\ \underline{+\ 30} \\ 42 \end{array}$ and to $\begin{array}{r} {}^{1}14 \\ \underline{\times\ 3} \\ 42 \end{array}$
9. AR–II–C–15 (Grade 7) 82% Multiply 2-Digit and 1-Digit Numbers Items 61–65 Given a two-digit number to be multiplied by a one-digit number, the learner will identify the product, with or without aids. SAMPLE $46 \times 3 = \square$	1. From the distractors it is difficult to identify which errors were "fact" and which were "place value" errors. The most obvious error came in renaming, e.g. adding 1 ten to 3 x 4 tens. 2. The distractors are not very well designed. Errors often observed such as writing the two products, e.g., 1218, adding before multiplying, e.g. 208, were not included. Hence, care must be exercised in judging the performance on these items — it may be higher than is warranted. 3. The objective is minimal.	

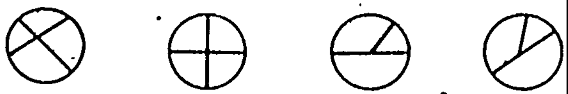
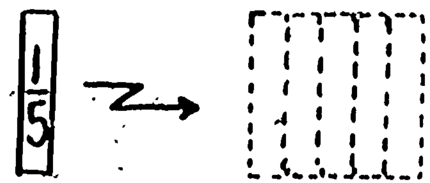
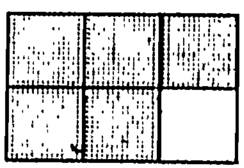
20

DIVISION OF WHOLE NUMBERS

OBJECTIVE / RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
10. AR-II-D-5 (Grade 7) 86% Supply Missing Factor/Multiplication Items 51–55 Given a sentence with one single digit, a missing factor, and a product (whole numbers), the learner will identify the missing factor. SAMPLE: $\square \times 6 = 36$	1. Thirty percent of the students missed at least one of the five basic division facts, stated in multiplication form. About 6 to 7 percent missed each of these two, $\square \times 6 = 36$ and $6 \times \square = 42$, making them the easiest of the five. From 13 to 15 percent missed each of these three, $4 \times \square = 32$, $9 \times \square = 54$ and $8 \times \square = 56$, making them the most difficult. 2. The most commonly chosen incorrect answer was a factor 1 more or 1 less than the correct one.	1. Relate $\div$ to x from the beginning of division instruction by asking the child to think "what times 6 is 42" and immediately asking, then $42 \div 6 = \square$, or $6\overline{)42}$ equals what? 2. Emphasize that finding a missing factor is dividing, and that whenever you have a product you can divide. For $5 \times \square = 40$, locate the product 40 in the 5 factor row of a multiplication chart. Go up the column above 40 to locate the missing factor, 8 at the top of the chart.
11. AR-II-D-7 (Grade 7) 80% Rewrite Division Fact as Multiplication Fact Items 71–75 Given a division fact, the learner will identify it rewritten as a multiplication fact. SAMPLE: Which multiplication sentence below is the inverse of $15 \div 3 = 5$? Answer: $5 \times 3 = 15$	1. The most common incorrect answer was the multiplication sentence formed by replacing $\div$ by x with the correct product shown, e.g., $15 \div 3 = 5$ becomes $15 \times 3 = 45$. 2. The word "inverse" probably accounted for most of the incorrect responses. The word is not used in many textbooks. The test items need to be reworded. A suggested rewording is, "If you know $15 \div 3 = 5$, then you know that?"... $3 \times 5 = 15$ would be be the correct response. 3. The objective is minimal with revision in wording of the test item.	3. When given a missing factor type of equation, have students write all related equations. If $6 \times \boxed{7} = 42$, then $\boxed{7} \times 6 = 42$, $42 \div 7 = \boxed{6}$, and $42 \div 6 = \boxed{7}$. Also, $\frac{1}{6}$ of $42 = \boxed{7}$ and $\frac{1}{7}$ of $42 = \boxed{6}$, may be included if this fractional form has been introduced. All these forms may be identified from an array. 

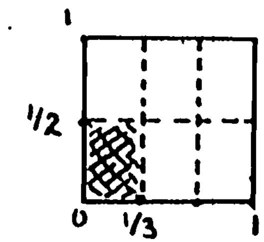
OBJECTIVE / RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
12. AR-II D 9 (Grade 7) 71% Division/One-Digit Divisor, Three Digit Dividend Items 76-80 Given a one-digit divisor (factor) and a dividend (product) of less than 100, the learner will identify the quotient (missing factor) if there is no remainder. SAMPLE $3\overline{)75}$	1. Two items were in horizontal form, e.g., 60 : 4 = ☐ . There was no noticable difference in performance between the two forms. 2. Almost half the pupils missed at least one of the test items. The percent incorrect for each item ranged from 17 to 24. Twenty percent missed 3 or more and 12% missed 4 or 5 of the test items. 3. These relatively poor results reflect the cummulative effect of errors in division facts (31 % missed at least one fact in objective 10), numeration (see objectives related to numeration), and to incorrect order of steps in the division algorithm. 4. The objective is minimal and the results are disappointing. The relation between division and multiplication – essential for learning the division facts and in checking division – has not been adequately learned by about one quarter of the students by the end of grade 6.	4. Look at suggestions for teaching numeration. Use them to identify the 45, in 4557 : 7 = ☐, as "45 hundreds". Then 45 hundreds : 7 is 6 hundreds with some hundreds left. 5. Before dividing by 2-digit or by greater numbers, try to get 80-90 percent mastery on 1-digit divisors. 6. Consider introducing and teaching the "short-division" algorithm. 7. With unsatisfactory results, overall, on division, consider a delay in grade placement of division by 2 or 3 digits. Spend more time and make a careful development up to and including objective 13.
13. AR-II-D 15 (Grade 7) 62% Division/One-Digit Divisor, Four Digit Dividend Items 81-85 Given an exercise with a dividend of four digits or less, and a one-digit divisor, the learner will identify the quotient. SAMPLE: 4557 : 7 = ☐	1. More than half of the students missed at least one item. The percent missing each item ranged from 23 to 35. Twenty-nine percent missed 3 or more items and 19% missed 4 or 5. 2. Three items were in the common form, e.g., $9\overline{)1989}$. There was no apparent difference between the results on the two forms. 3. $4\overline{)1632}$ was the only item that had a zero in the tens place of the quotient; 25 % got this item incorrect. 4. The objective is minimal.	

FRACTIONS–MEANING

OBJECTIVE / RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
14. AR–III–A 1 (Grade 7) 65% Identify Congruent Parts Items 196–200 Given several objects, some divided into congruent parts, some divided into noncongruent parts, the learner will identify congruent parts. SAMPLE: Which figure is divided into congruent parts?  F G H J	1. The term congruent may have been unfamiliar to many students. Twelve percent of the students got zero correct answers. 2. The fact that only 65% of the students had success with this objective is disappointing. The objective should be revised so that the emphasis is placed on recognition of equal size parts of unit regions. With revision in wording, the objective is minimal. 3. The term congruent should receive more attention in the intermediate grades in connection with geometry topics.	1. Teach the term congruent in grades 4-6 by using activities like tracing plane figures with pencil and tracing paper. 2. Emphasize activities which stress the identification of the unit region. For example, show a picture of part of a rectangular unit region and request that the student reconstruct (through drawing) the unit. 
15. AR–III–A 18 (Grade 7) 76% Identify Shaded Area of Figure with Fraction Items 111–115 Given a diagram divided into congruent parts, with some parts shaded, the learner will identify the shaded area by identifying an appropriate fraction. SAMPLE: Choose the fraction that names the shaded part.  A $\frac{1}{5}$ B $\frac{1}{6}$ C $\frac{5}{6}$ D $\frac{7}{8}$	1. From 7 to 10 percent of the students chose the fraction resulting from comparing the number of white parts with the total number of parts. 2. Choosing the fraction naming the comparison of shaded to white parts attracted from 8 to 14 percent of the students. 3. This result is most disappointing. By the end of grade 6, students should have mastered this fundamental concept. Much of the subsequent computational work with fractional numbers depends on this objective. The objective is minimal.	3. Stress the interpretation of the fraction symbol $\frac{3}{5}$ which means that the unit has been divided into 5 parts of the same size and three of these parts are being considered (shaded, pieces of pie eaten, etc.). 4. Teach the meaning of the word "shaded" prior to the unit on fractions. 5. Review the concept of order from least to greatest with whole numbers prior to working with fractional numbers.

OBJECTIVE RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
16. AR–III–A–19 (Grade 7) 56% Order Fractions with Like Denominators Items 106–110 Given any five fractions with like denominators, in random order, the learner will identify them in order (halves, thirds, fifths, sixths, eighths, tenths); with or without the use of aids. SAMPLE: Which group of fractions below is in order from *least to greatest*? F $\frac{4}{5}, \frac{6}{5}, \frac{3}{5}, \frac{8}{5}, \frac{2}{5}$ G $\frac{2}{8}, \frac{4}{8}, \frac{3}{8}, \frac{6}{8}, \frac{9}{8}$ H $\frac{1}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{7}{6}$ J $\frac{3}{10}, \frac{8}{10}, \frac{6}{10}, \frac{9}{10}, \frac{5}{10}$	1. Twelve percent of the students got zero items correct. The most popular distractor was the numbers in order from greatest to least. 2. Only 64% of the students could identify whole numbers in order from least to greatest by the end of grade 3. See objective 17 on the grade 4 test. 3. The MCTM/MEAP Conference indicated that the terminology least to greatest is a problem area. The writers feel that the objective is minimal and this terminology should be mastered by the end of grade 6.	6. Use five 3 x 5 cards as unit regions and color $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$, and $\frac{5}{5}$ of each card respectively. Have the students say how much is colored on each card and arrange the cards from least to greatest left to right on their desks.

FRACTIONS-MULTIPLICATION

OBJECTIVE	RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
29. AR-III D 1 (Grade 4) Identify Fractional Part of Set of Objects Items 151-155 Given groups of less than 13 objects and a statement ($\frac{1}{2}$ of, $\frac{1}{4}$ of,) the learner will identify the group that shows the appropriate fractional part of the whole. SAMPLE: Which picture shows $\frac{1}{4}$ of 8? Choices are pictures with objects in rectangular arrays a ring drawn around some of the objects in each array.	15%	1. By far the poorest results on either test was obtained on objective AR - III D 1. Interpreting $\frac{1}{4}$ of 8 in a symbolic-pictorial situation is very difficult for most students in primary grades. 2. A picture showing the same number of objects encircled as the denominator was available on item 153 and 155. For $\frac{1}{3}$ of 12, on item 155, 46 percent selected the picture with 3 objects encircled. The percent selecting this type of distractor on Item 153 was 57. 3. The writers recommended that if objective AR III D 1 remains in the K 3 sequence, then it might better be tested using concrete objects. "show me one-fourth of these eight crayons".	1 The emphasis on objective AR III D 1 should be on using concrete objects to develop the notion that $\frac{1}{a}$ of N means (N ÷ a). Example: show the student 15 blocks and ask him to hand you one-third of these blocks. The student should be taught to separate the 15 blocks into 3 sets, each with five blocks. Ask, "How many blocks in each of these 3 sets?" 2. Students should understand that multiplying a number that is less than one results in a product which is smaller than the multiplicand. For example Will ($\frac{1}{2}$ x $\frac{1}{3}$) be greater than $\frac{1}{3}$? Answer No, it will be smaller. . . you're multiplying by one half. Notice that the erroneous algorithm $\frac{1+1}{2 \times 3}$ produces the answer $\frac{2}{6}$ which equals $\frac{1}{3}$
21. AR III D 4 (Grade 7) Multiply 2 Unit Fractions Items 136-140 Given two unit fractions with denominators less than seven, the learner will compute the product with or without the use of a model. SAMPLE: $\frac{1}{6} \times \frac{1}{5} = \square$ Answer: $\frac{1}{30}$	70%	1. For the four items where adding numerators and multiplying denominators was a choice, the percent selecting this incorrect response ranged from 10 to 12. The percent selecting the choice where adding denominators was shown ranged from 5 to 10. These were the two basic wrong answers offered and they accounted for the majority of the incorrect responses. 2. Two items were given in a vertical format and 3 in horizontal form. There was no apparent difference in performance. 3. Many students could be obtaining the correct results for each item without understanding the operation of multiplication with fractional numbers. 4. The objective is minimal.	3. A good understanding of the multiplication algorithm for fractional numbers can be built on the idea of determining area by multiplying the two dimensions. The dimensions of the unit region are 1 by 1. The dimensions of the shaded region are $\frac{1}{2}$ by $\frac{1}{3}$. The shaded area is $\frac{1}{6}$, that is, 1 out of 6 congruent parts of the unit region. 

OBJECTIVE / RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
17. AR–III-B-4 (Grade 7) 65% Identify Sum Of 2 Fractions With Like Denominators Items 121–125 Given two common fractions with like denominators and a sum greater than 1, the learner will identify the sum, with or without the use of fractional cut-out parts. SAMPLE: $\frac{2}{3} + \frac{4}{3} = \square$ Answer $\frac{6}{3}$	1. Fifteen percent of the students got zero items correct. The poor performance on objectives 14 and 15, fraction representation, makes these results not so surprising. This is very alarming considering that objective 17 is initially developed about Grade 4 and is of primary importance to subsequent computation topics. 2. There was no apparent difference in performance between vertical and horizontal (number sentence) formats. 3. The most popular distractor involved adding the demoninator, e.g., $\frac{3}{4} + \frac{5}{4} = \frac{8}{8}$. The percents selecting this wrong choice ranged from 18 to 20. The fact that such a common misconcept as adding denominators should persist through sixth grade is extremely disappointing. If it were not for this error, the results on this objective would be at a more acceptable level of near 85 to 90 percent. 4. Directions in the test booklet should note that answers need not appear in mixed or reduced form. 5. The objective is minimal.	1. The foundation program for fractional numbers must receive more attention in grade three and be maintained continuously through grades 4, 5, 6 and 7. The low level of performance on objectives 17-20 can be connected to the lack of attainment of objectives 14-16. 2. Students who are adding denominators need to use concrete materials. Strips of paper can be used as parts of the unit. To add $\frac{2}{3}$ and $\frac{4}{3}$, the student can count out one-, two-thirds in one pile and one-, two-, three-, four-thirds in another pile. Putting the two piles together; he counts one-third, two-thirds, ... six-thirds in all. 3. Similarly, for subtraction the student can be presented with 6 unit strips of paper and when asked, he can demonstrate $6 - 1 = 5;$ $1 = \frac{3}{3}$ and $\frac{3}{3} - \frac{2}{3} = \frac{1}{3}$. Finally $6 - \frac{2}{3} = 5\frac{1}{3}$
18. AR–III–B–7 (Grade 7) 61% Add 2 Mixed Numbers With Like Denominators Items 126–130 Given two mixed numbers with like denominators, the learner will write the sum. SAMPLE: $4\frac{1}{3} + 2\frac{1}{3} = \square$ Answer $6\frac{2}{3}$	1. Item 129 required $4\frac{4}{4}$ to be renamed as 5 and Item 130 required $5\frac{11}{8}$ to be renamed as $6\frac{3}{8}$. The students did not do as well on these two items as on the other three items. 2. Only Item 126 has adding the denominators as a distractor; sixteen percent responded with this error. If similar distractors had been available for other items, the overall results probably would have been lower.	

OBJECTIVE	RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
		3. The results on both Objectives 17 and 18 are disappointingly low. The results are probably indicative of a lack of maintaining this skill during the fifth and sixth grades as well as inadequate conceptual work in grades 3 through 5. 4. The objective is minimal.	
19. AR–III–C–4 (Grade 7) Subtract Fraction From Mixed Number/Like Denominators Items 131–135 Given a mixed number and a fraction with like denominators of 2, 3, 4, 6, or 8, where no regrouping is necessary, the learner will find the difference. SAMPLE: $3\frac{5}{6} - \frac{1}{6} = \square$ Answer $3\frac{4}{6}$	63%	1. The students do not find the number sentence format more difficult than the vertical format. 2. Items 131 and 132 had the sum as one of the choices. This distractor was chosen by 14 to 16 percent and may have been influenced by the fact that the previous five items were addition problems. 3. The objective is minimal.	
20. AR–III–C–6 (Grade 7) Subtract Common Fraction From Whole Number Items 116–120 Given a whole number and a common fraction with a denominator of 2, 3, 4, 6, or 8, the learner will find the difference with or without the use of fractional parts. SAMPLE: $6 - \frac{2}{3} = \square$ Answer $5\frac{1}{3}$	43%	1. Fifteen percent of the students got zero items correct and 43 % missed 3 or more. 2. In general, the most frequent distractor selected for this set of items (15 to 20 percent) involved adding instead of subtracting; e.g., $8 - \frac{5}{6} = 8\frac{5}{6}$. 3. On Items 117 and 118 there is evidence that many students (about 22 %) decreased the whole number by 1, but the operation was not formed in their understanding well enough to complete the subtraction. For example, $6 - \frac{2}{3} = 5\frac{2}{3}$. 4. The objective is minimal.	

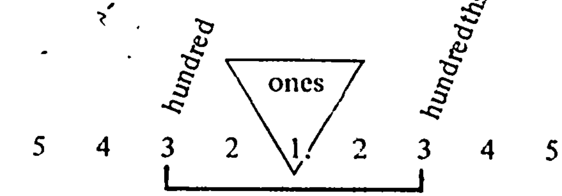
OBJECTIVE RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
22. AR–IV–A–3 (Grade 7) 84% Identify Illustrated Decimal Fraction Items 96–100 Given a model of a fraction illustrating hundredths, the learner will identify the decimal fraction as illustrated. SAMPLE: Pictures of ten-by-ten rectangular arrays were used. The directions were to choose the decimal that names the shaded part of each figure.	1. A lack of effective distractors resulted in an artifically higher level of attainment for this objective. For example, where the correct response was .53, choices such as 5.3 or .053 were not available. Students had only to count the number of shaded squares; the decimal point could have been overlooked.	1. Place value concepts appear to need more attention in the total curriculum, grades 1-6. Much of the work with decimal fractions is being delayed until sixth and seventh grade; this is too late in the program to begin this development. 2. Money cannot be depended upon to help build an understanding of tenths and hundredths unless the transfer is strengthened through instruction. If one dollar is related to one then one hundred cents in a dollar can be related to $\frac{1}{100}$ and to .01. Example: 5 cents can be written as \$0.05 because the .05 means 5 hundredths. and \$0.05 means 5 out of the 100 cents in one dollar. 3. The hundred square is a graphic model of the decimal fractions, tenths and hundredths. A student touches 1 square and has touched .01 of all the squares. If he runs his finger along 10 of the squares, he has touched .10 or .1 of all the squares. What decimal fraction should he write if he touches $\frac{1}{2}$ of the hundred squares? (.5 or .50)
23. AR–IV–A–7 (Grade 7) 26% Name Place Values of Decimal Fraction Items 101–105 Given a decimal fraction of no more than three places, the learner will name the place value of each digit, without aids. SAMPLE: .437 means which of the following? Answer: 4 tenths, 3 hundredths, 7 thousandths.	1. There were two items to hundredths. The most popular distractor involved having the place value names in reverse order. For example: .74 means 7 hundredths, 4 tenths. On Item 102, 40 percent of the students interpreted .91 as 9 tens, 1 one; ignoring the decimal point completely. 2. It is not clear to what extent the students confused thousands with thousandths. The major distractor for the three items dealing with thousandths was the reverse order of the place value names. For example, on Item 104, 41 percent selected 6 thousandths, 5 hundredths, 8 tenths for the meaning of .658. 3. Forty percent of the seventh graders did not get any of the five items correct on objective AR–IV–A–7. This objective is not receiving appropriate attention in the grades 3-6 instruction program.	4. The concept of place value is fundamental to the understanding of decimal fractions. Use place-value strips above the chalkboard. To extend these downward to the right to include the decimal fraction places, start with 1000 and repeatedly divide by 10 until .001 is reached. Stress the numerators are shown by the numerals, and the denominators are indicated by the last used place.

OBJECTIVE	RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
		4. An alternative format for testing would be "In .437, the 3 means which of the following? A. 3 tens B. 3 tenths C. 3 hundredths D. 3 thousandths 5. The objective is minimal.	(see below)

TEACHING AND CURRICULUM SUGGESTIONS

thousands	hundreds	tens	ones	tenths	hundredths	thousandths
1	0	0	0			
	1	0	0			
		1	0			
			1			
				.1		
				.0	1	
				.0	0	1

5. The symmetry of the place values should be connected to the place value names.



a. The decimal point indicates the place of the ones. The ones place is the middle place in this symmetry.
b. The 3's are in the hundreds and hundredths places respectively from left to right.
c. Each place is $\frac{1}{10}$ as much as the place at its immediate left.

$$1000 \quad 100 \quad 10 \quad 1. \quad \frac{1}{10} \quad \frac{1}{100} \quad \frac{1}{1000}$$

$$\frac{1}{10} \times 1000 = 100 \qquad \frac{1}{10} \times \frac{1}{100} = \frac{1}{1000}$$

OBJECTIVE	RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
			6. To encourage the students in obtaining a feeling for the comparison between 100, 1 and .01, cut construction paper up into the following sizes: 1 meter by 1 meter square; 1 decimeter by 1 decimeter square; and 1 centimeter by 1 centimeter square. Let the decimeter square represent 1. Cut several copies of each size square and let the students represent numbers like 203.05 by counting out and arranging the squares under each numeral.
		DECIMALS—ADDITION AND SUBTRACTION	
24. AR–IV–B–3 (Grade 7) Addition/Subtraction Decimal Problems: Tenths Items 141–145 Given a verbal problem involving addition and subtraction of decimal numbers involving only tenths, the learner will find the answer. SAMPLE: It rained .3 inches one week, and .6 inches the next week. How much did it rain all altogether?	75%	1. The level of performance on this objective is probably artificially high. Students add or subtract without paying attention to the decimal point on four of the five items. On Item 145 (2 + 1.7) thirty-three percent of the students chose 1.9 as the answer instead of 3.7. The percent correct on the other items ranged from 86 to 88. 2. Reading did not appear to be a difficulty with these items. Incorrect answers seemed to be the result of arithmetic errors, not reading errors. 3. There was no difference between addition and subtraction problems as to level of difficulty. 4. Misconcepts such as .9 + .4 = .13 and .5 + 4 = .09 were not adequately measured because the choices were not available. 5. The objective is minimal.	1. Addition and subtraction using decimal notation is often neglected in the fifth and sixth grades. The material should be taught in grades 4 through 6 with the initial concept development starting in grade 3. 2. Place value plays a fundamental role in the overall ability to add and subtract using decimal notation. Students should be able to represent 2.354 as $2 + \frac{3}{10} + \frac{5}{100} + \frac{4}{1000}$ and to rename each fraction: $2 + \frac{300}{1000} + \frac{50}{1000} + \frac{4}{1000} = 2\frac{354}{1000}$ 3. Review computing sums using fractions with denominators of 10, 100 and 1000. Examples: $5\frac{3}{10} + 6\frac{4}{10} = 11\frac{7}{10}$ $4\frac{1}{10} + 8\frac{1}{100} = 12\frac{11}{100}$

DECIMALS—ADDITION AND SUBTRACTION (continued from page 32)

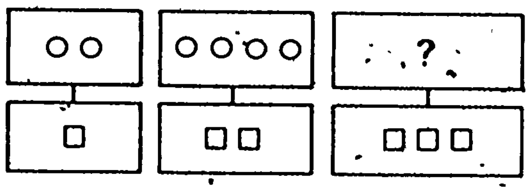
OBJECTIVE	RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
25. AR–IV–B–6 (Grade 7) Addition/Subtraction Decimal Problems: Tenths Items 146–150 Given a verbal problem involving addition and subtraction of decimal numbers involving tenths and hundredths, the learner will find the answer. SAMPLE: Gloria ran .56 miles, Sue ran .3 miles. How much farther did Gloria run than Sue?	14%	1. Eighty-one percent of the students were correct on Item 146 which involved addition where the decimals were aligned— .46 + 53. However, only 15 percent were correct on Item 147 which was a subtraction with ragged decimals— .56 – .3. Seventy percent chose the incorrect response .56 – .3 = .53. 2. The students performed better on the two addition problems (81% and 40% correct) than on the three subtraction problems (68%, 20% and 15% correct). They performed better on those problems where the two numbers were expressed to the same place value (add = 40%, subt. = 20% and 15% correct). 3. Reading difficulties probably did not account for very much of the low performance since there was similar reading to be done with objective 24 where the results were better. 4. The objective is minimal.	4. Use a number line graduated in tenths to interpret adding with numbers like 0.3 + 1.2 = 1.5. Emphasize problems such as 3 – .6 = 2.4 where the sum is a whole number and one addend is a number of tenths. 5. Do not allow your major work with decimals to become connected with the metric system before the students have mastered some of the metric concepts apart from decimals. 6. Stress having some feeling for the reasonableness of the answer (approximation) by making sure that the students are familiar with magnitude of the amounts under consideration. For example 8 – 1.6 ≠ 1.2 because 1.6 + (some number) must equal 8. Since 1.6 is about 2, then the answer is about 6. 7. For problems involving "ragged decimals", for example, 2.1 + 3 + 4.05, teach the students to rewrite the addends before computing. Thus, 2.1 + 3 + 4.05 becomes 2.10 + 3.00 + 4.05. 8. Give practice in transferring addition problems from work or equation form to vertical form with the decimal points aligned.
26. AR–IV–B–9 (Grade 7) Addition/Subtraction Decimal Problems: Vertical* Items 91–95 Given an addition and subtraction decimal problem in horizontal or vertical form with no more than five (5) digits and no more than three (3) decimal places, the learner will find the sum or difference. SAMPLE: 1.3 + .52 + .037 = ☐ *Horizontal format was also used	37%	1. Vertical format appears slightly easier than horizontal. This is probably because in the vertical format, the decimals are aligned. 2. Students did better on the addition problems. They did much better on the addition in vertical form than the subtraction in horizontal form using "ragged decimals". 3. There seems to be very little difference between the performance on comparable problems between objectives 25 and 26 where the decimals are aligned. Where the decimals were "ragged", the students performed better with the operation than they did with the word problems. 4. The objective is minimal.	

OBJECTIVE / RESULTS

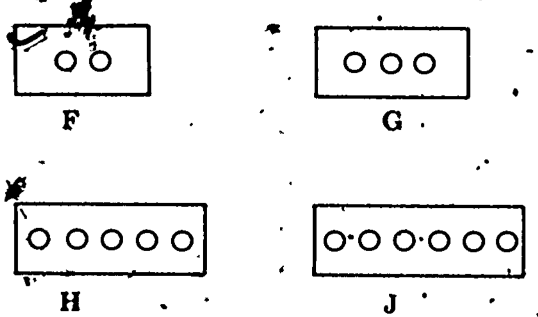
27. AR–VI–15 (Grade 7) 38%
Identify Par of Sets/Equivalent Ratio
Items 176–180

Given a picture of sets paired in (A) a one-to-one, (B) a many-to-one, or (C) a one-to-many and part of another pair, the learner will identify the pair that keeps the ratio equivalent.

SAMPLE:



Which group of circles below completes the pattern shown above?



REMARKS

1. The test items do not measure understanding of equal ratios. The stem was probably interpreted in several ways by the students since it only refers to completing a pattern.

2. Items 179 and 180 are "one-to-many" items. Thirty-one and thirty-four percent respectively chose the response which indicates they were completing a sequence pattern like 1, 3, ?. The answer is 5 because 3 – 1 = 2 and 3 + 2 = 5. For the two "one-to-one" items, the misconcept of adding on to complete the pattern results in a correct answer.

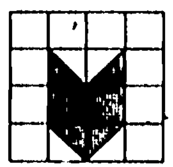
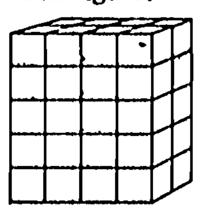
3. The object and the test items need revision. As stated and tested, the expectation is not minimal. However, the ideas of ratio as suggested are important and should be learned by Grade 7.

TEACHING AND CURRICULUM SUGGESTIONS

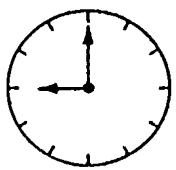
1. Ratio concepts are not developed very well in the various text series. An understanding of ratio and proportion can make a valuable contribution to problem solving.

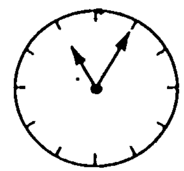
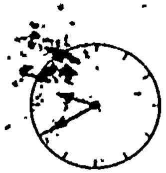
2. Use some real life ideas to teach the concept of equal ratios. For example, the ratio which compares the number of cans of frozen juice concentrate with the number of cans of water is one to three, or $\frac{1}{3}$. To keep the same taste, four cans of frozen juice must be mixed with 3 x 4, or 12 cans of water. Strengthen the connection between equal ratios and the generalization, $\frac{a}{b} = \frac{n \times a}{n \times b}$.

OBJECTIVE / RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
30. M I A 5 (Grade 4) 84% Measure Objects To Nearest Inch Items 156–160 Given rulers specially scaled in inches and half-inches, the learner will identify the measurement of objects to the nearest inch. SAMPLE: A picture of a ruler, half-inch unit, under an everyday object, e.g., candy bar, nail, etc. "About how many inches long is this candy bar?"	1. The range of correct responses was 85 to 89 percent on four of the five items. Students who selected the answer nearest the end of object on the test paper would score high on on these items. On Item 159, sixteen percent chose response H. 3 at the end on the pencil The correct response was J. 4 and only 75% chose it. If real rulers were used, the results might be somewhat less than 84 percent. 2. The objective is minimal.	1. Many of our students do not have an adequate grasp of some of the fundamental concepts of measurement. Many students apparently cannot identify the unit being used in a particular measurement. The notion of identifying and counting a unit in a measurement should be introduced earlier in the primary years and thoroughly maintained on into the middle school years. 2. Specific instruction in reading scales should be given in grades 3-6. Students should be taught to read measures from such everyday scales as thermometers, speedometers, gas gauges, and pressure gauges as well as rulers. Emphasis should be placed on how to assign a number to a measure when the reading is not directly indicated on the scale. 3. Estimation should be stressed with all measurement. After estimating the lengths of many objects pencil, paper, string, ribbon, paper clip, etc., children measure to see how close their estimates are to the actual ruler measurement. Have students closely estimate the length of many pictured small objects and measure to the nearest quarter inch. Identify the term quarter-inch with $\frac{1}{4}$ inch. 4. The assessment items (and to some extent the objectives) can be faulted in that concrete objects and real measurement tools are not stressed. These things should be used in the classroom. Install a measurement laboratory in one corner of the classroom and make actual measurement activities a regular part of the mathematics program.
28. M I A 10 (Grade 7) 53% Measure Objects To Nearest Quarter Inch Items 166–170 Given rulers specially scaled in quarter inches or eighth inches, the learner will measure objects to the nearest quarter unit. SAMPLE: A picture of a ruler (three of the five were in quarter-inch units and two were in eighths) under an everyday object, e.g., pencil, paper clip, etc. "What is the length of this paper clip to the nearest $\frac{1}{4}$ inch?"	1. Unlike the grade 4 test items, the students here did not appear to go for the answer nearest the end of the object in the picture. 2. This is a disappointing result especially when compared with the results at grade 4 on objective M I A 5. It appears that linear measure is not taught in grades 4-6 with the view of using fractional parts of units. 3. It is difficult to discern from the data whether the students cannot estimate to the nearest quarter inch or whether they do not completely understand the notion of a unit of length. In Item 168 they appear to have perceived the end of the ruler to be a whole unit and 20 percent responded $2\frac{1}{2}$ inches instead of the correct $2\frac{1}{4}$ inches. In Item 170, 28% of the students appear to have perceived the small divisions, $\frac{1}{8}$ in., to be $\frac{1}{4}$ inches and they counted two of these divisions to answer "$5\frac{2}{4}$" inches instead of $5\frac{1}{4}$.	

OBJECTIVE	RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
		4. On two of the items where the ruler is graduated in fourths, the performance was greater than on the two items where the ruler is graduated in eights. Children need specific help in interpreting the meaning of the marks of varying length on the ruler. 5. The objective is minimal for grades 4-6 and more emphasis should be placed on the actural use of rulers in concrete situations.	5. Geoboards and tangrams can be used to give the students some highly motivational and concrete experience with units of area. Children can also build solid figures out of wooden inch blocks and determine volume.
29. M–I–B–8 (Grade 7) Estimate Area of A Polygon Items 211–215 Given a polygon, the learner will estimate its area in square units. SAMPLE: Estimate the area of each figure in square units. 	49%	1. The common incorrect response for these items involved counting parts of the unit as whole units. Nineteen percent chose 6 as the answer to the item shown in the sample. 2. Only 49% attained this objective and 27% got no more than one of the five items correct. The five test items did measure the basic concept of area and it is obvious that this area concept is not receiving adequate attention in grades 3-6 that it should. 3. The objective is minimal.	6. Students need special practice in space visualization and interpretation of drawings. It should not be assumed that students can view a 2-dimensional drawing and visualize what the solid figure would look like without adequate instruction.
30. M–I–C–7 (Grade 7) Name Number Of Units In A Rectangular Solid Items 216–220 Given a drawing of a rectangular solid divided into units (dimension less than or equal to 5 units), the learner will name the number of units in it. SAMPLE: This is a cubic unit.  Choose the numeral that tells how many cubic units are in each figure. 	28%	1. A significant proportion of the students did not perceive the cubic unit. The 2-dimensional drawing does present some difficulty. From 38 to 45 percent counted all of the observable two-dimensional units, i.e., the observed surface area. 2. Twenty-eight percent attaining this objective is a discouraging figure. The fact that 55 percent got no more than one item correct indicates a real problem in the curriculum. 3. The objective should be tested with real objects.	

TIME

<table>
<tr><th>OBJECTIVE</th><th>RESULTS</th><th>REMARKS</th><th>TEACHING AND CURRICULUM SUGGESTIONS</th></tr>
<tr><td>31. M II A -6 (Grade 4)
Telling Time
Items 111 -115
Given the reading" o'clock" and a clock face, the learner will identify the clock showing the appropriate time.
SAMPLE: Which clock shows 9 o'clock? answer
</td><td>89%</td><td>1. While it is not a major concern, it should be noted that only 75% of the students got all five items correct. There were no numerals on the clock face and it may be that some of the students had difficulty counting the unit-hour intervals. The popular incorrect responses were generally within one hour of the correct response.</td><td>1. There are several complexities in reporting time. Students must discriminate between the two hands, tell "before" from "after", utilize a variety of reporting notation, e.g., 2:30, two-thirty, half past two, etc.

2. Students need to practice more with time oriented problems in order to become more proficient. Incorporate more problems involving time situations into the instructional program, For example,

(a) What time is it now?
(b) In 2 hrs. and 35 mins. what time will it be?</td></tr>
<tr><td>32. M -II-A 10 (Grade 7)
Tell Time
Items 181-185
Given a clock face with hands on it, the learner will choose the correct time notation.
SAMPLE: What times does this clock show?
A. 9:20 B. 9:35
C. 9:40 D. 10:40
</td><td>59%</td><td>1. Performance on three of the five items was low. On these items, the students selected incorrect choices which indicate that they were confused as to whether the minute hand was before or after the hour. For example, on Item 184 the correct response was 2:55. Sixteen percent selected 3:05 (probably thinking 5 minutes before 3) and eighteen percent selected 3:55 (fifty-five minutes after the wrong hour).
2. Confusing the hour hand with the minute may have been the reason why 16% of the students chose 11:15 as the answer for Item 184. The correct answer was 2:55. There was not enough evidence available to determine if this confusion occurred on the other items.
3. While some would argue that this is not a mathematics item, we believe that the concepts involved are primarily the responsibility of the mathematics program. The fact that only 59% of the students attained this objective is a matter of concern.
4. The objective is minimal.</td><td>3. Performance on objective 32 should improve once students are taught the meaning of A.M. and P.M.</td></tr>
</table>

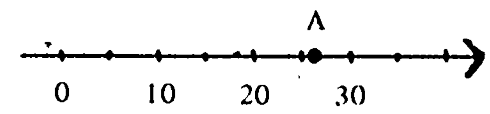
OBJECTIVE / RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
32. M–II–A–11 (Grade 7) 68% Identify A.M. and P.M. Items 186–190 The learner will use A.M. and P.M. notation in writing time. SAMPLE: Which one of these means two o'clock in the afternoon? F. 2:00 A.M. G. 2:00 P.M. H. 12:00 A.M. J. 12:00 P.M.	1. Many students were confused by these five items. While the objective does not appear to be one of crucial significance, the results reveal that students have difficulty computing with the modular arithmetic of "A.M." and "P.M.". 2. It appears that some of the students used A.M. for P.M. and vice versa. The range of percents for correct responses was 69-81. The range of percents selecting the correct hour but using the incorrect A.M. or P.M. notation was 10-19. 3. The objective is minimal.	
	MONEY	
32. M–II–B–3 (Grade 4) 75% Identify Greatest or Least Amounts of Money Items 176–180 Given three to five different amounts of money, all less than or equal to $5.00, the learner will identify the greatest or the least. SAMPLE: Which is the <u>least</u> amount of money? F. $2.15 G. $2.05 H. $2.51 J. $3.15	1. The range of percent correct per item was 68-88. Three formats were used: (1) pictures of money, (2) word names for money, e.g., dollar, and (3) dollar notation, e.g., $2.98. There was no apparent pattern in performance favoring any one of these formats. Students also performed as well with <u>greatest</u> as with <u>least</u>. 2. Eighty-eight percent correctly selected the picture of the half dollar as the <u>greatest</u> amount of money. This could be an artifically high result since the half dollar was also the largest coin shown. 3. While 86% correctly identified the word "dollar" as the greatest amount of money given, only 68% correctly identified the picture showing 2 dollar bills as the greatest amount of money. On this latter item, 22% selected the only choice which did not use a dollar bill. Perhaps because the picture showed the greatest number of coins. 4. The objective is minimal. While the performance is low, it is comparable with objective 15, value of coins - 78% attaining, and objective 17, least to greatest - 64% attaining. The low performance on objective 32 may be more related to the terms <u>least</u> and <u>greatest</u> and less to knowledge of money.	1. There is apparently not enough concrete experience in the classroom with money and money notation. 2. To review the different money notations have students match cards, e.g., $20 $20.00 twenty dollars or 5¢ $0.05 five cents 3. Students should be given more opportunities to make approximations with amounts of money. Simple menus are posted. Students become waiters, make out orders and total the amounts. They become cashiers and make correct change when a customer pays his bill. They become customers who may spend up to only a specified amount and must learn to estimate what their final bill will be.

MONEY (continued)

OBJECTIVE	RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
33. M–II–B–7 (Grade 7) Add/Subtract: Money Items 26–30. Given two money values, the learner will add or subtract using dollars and cents notation. SAMPLE: $2.78 − .27 $37.50 −25.85	75%	1. The results on these items is comparable to the results on the subtraction items, objective 5. The difficulty does not lie with the money notation, but rather with a weakness in subtraction. 2. Some of the students may have mixed the two operations and subtracted when addition was requested. 3. The objective is minimal.	4. Small (spring or fall) type catalogues are consulted by students who may spend up to but not exceed ~~$20.00~~ (or any given amount). As they select items they wish to buy, they write a rounded amount beside the listed price, to help approximate their spending.
34. M–II–B–8 (Grade 7) Approximate Sum/Difference: Money Items 31–35 Given two money values, the learner will choose the approximate sum or difference to the nearest dollar or ten cents. SAMPLE: $1.89 −1.52 answer $.40	62%	1. Students do not appear to have firm grasp of approximation. On Item 32, for example, 22% responded with $1.60 as the answer to $1.69 rounded to the "nearest dime". 2. The vocabulary of "sum" and "difference" was probably of negligible difficulty for the students because problems also had + or − to show the operation. 3. The objective is minimal.	5. Use grocery ads from papers to compare prices, buy from a shopping list and keep within a food budget for a family.
35. M–II–B–9 (Grade 7) Solve Problem with Money Values Items 151–155 Given verbal problems consisting of one or two operations involving money values less than or equal to $20, the learner will solve the problems. SAMPLE: Jan went to the Post Office to buy stamps. If she bought 20 eight-cent stamps and gave the clerk $5.00, what was her change? answer $3.40	49%	1. Solutions to two-step word problems represent a very difficult area in the curriculum. The major difficulty is not with the fact that the student is dealing with a verbal problem. Nor is it primarily due to instability with computation. The computation of verbal presentation and two mathematical operations appears to be too complex for the majority of grade 4-6 students to handle. 2. Poor performance may be due to lack of attention to word problems in the mathematics program. 3. While the writers feel that the grade 4-6 program should strive to have students mastery this type of problem, we are uncertain whether mastery is reasonable for 85% of the pupils.	

ERIC Full Text Provided by ERIC

TEMPERATURE

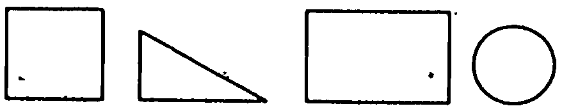
OBJECTIVE / RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
33. M–II–C–1 (Grade 4) 71% Identify Temperatures Items 166–170 Given a Fahrenheit or Celsius thermometer, the learner will identify the temperature (above zero) to the nearest degree. SAMPLE: Picture of thermometer given-scale markings varied from item to item. "What is the temperature to the nearest degree?"	1. The level of performance was 84 to 86 percent correct on two items where the picture showed the reading to be at a numbered scale marking. Eighty-eight percent got Item 168 correct, but there was no other realistic choice available among the distractors. 2. Some of the sudents may not have understood the terminology "to the nearest degree". Poor results on Items 166 and 169 (34% and 73% correct respectively) indicate that the students had difficulty in reading a scale marking between two numbered positions. 3. The objective is minimal. Students need more experience reading measurement scales on actual measurement tools and drawings of various scales.	1. Have students read indoor and outdoor temperatures on real thermometers. 2. Emphasize activities where the students must give the number which is "nearest" the point on the number line. For example, "Is point A nearest 30 or 25?"  3. Construct a large cardboard thermometer with a red and white movable tape. The Fahrenheit scale may be written on one side of the thermometer and the comparable Celsius scale on the other. Each day a different student pulls the tape to the predicted daily high temperature and reads one scale and/or the other. For K-3 cover one scale while a child reads the other, or have two thermometers. Scales should be given in 2° intervals. Have actual thermometers for follow-up practice.
36. M–II–C–3 (Grade 7) 75% Read Temperatures Items 171–175 Given a Fahrenheit or Celsius thermometer, the learner will identify temperatures to the nearest degree, using the degree (°) symbol. SAMPLE: Picture of thermometer given – scale markings either by ten or by five with no unlabeled scale markings.	1. The results were above 80% correct on three of the five items. On the other two items the most frequently chosen distractor indicates a weakness in estimation. For example, 74% correctly estimated 35° on Item 173 and 16% felt the answer was 32°. 2. The results for these two objectives indicate very little growth between grades 4 and 6. 3. The objective is minimal.	

OBJECTIVE — RESULTS

34. G–I–A–2 (Grade 4) 74%
Identify Geometric Shapes
Items 86–90

Given pictures of various shapes, the learner will identify circles, triangles, squares, and rectangles as requested.

SAMPLE: Which figure is a triangle?



A B C D

REMARKS

1. Only 68% of the students correctly identified a rectangle.
2. A common incorrect response for identifying a triangle was a rectangle.
3. Ninety-one percent correctly identified a circle. Also, the frequency of incorrectly identifying a circle for a square, triangle or rectangle was negligible.
4. The objective is minimal.

TEACHING AND CURRICULUM SUGGESTIONS

1. While it could be argued that it is not crucial that nearly all students be able to recognize suggestions of planes by grade 7, the fact that only 73% could do this indicates a lack of attention to geometry this is crucial.

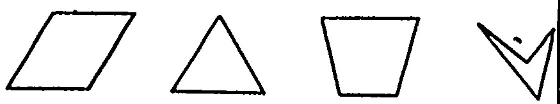
   The study of certain geometry topics can make a positive contribution to the overall attainment of skills by elementary and junior high students. Not only is the geometry itself essential, the various concepts can give meaning to and reinforce certain arithmetic concepts. Parallel and perpendicular are used in graphing; rectangular arrays are used in multiplication; and the union of geometric properties with arithmetic operations gives the student some very powerful problem-solving tools.

OBJECTIVE — RESULTS

37. G–I–A–4 (Grade 7) 66%
Name Quadrilaterals
Items 191–195

Given a set of quadrilaterals, the learner will identify and name a parallelogram, a square, and a rectangle.

SAMPLE: What figure is a parallelogram?



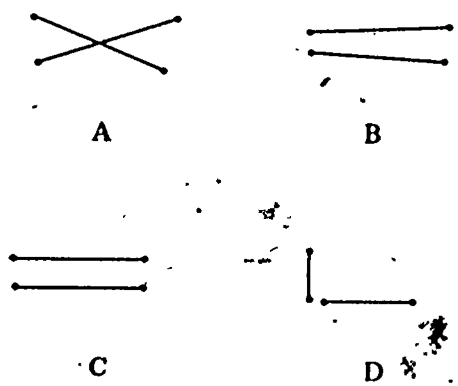
A B C D

REMARKS

1. Ten percent of the students failed to select the square from a set of other quadrilaterals. This shows a small gain in the ability to recognize the square shape when compared to 15% of Grade 4 students who failed to select the square from a somewhat simpler set containing a circle, a triangle and a rectangle.
2. Twenty-two percent of the Grade 7 students did not know the name of a rectangle when given the shape and in another item 29% failed to identify the shape when given its name.
3. Thirty-three percent made incorrect answers when asked to select a parallelogram and in another item 25% did not know its name.
4. The objective is minimal. The poor Grades 4 and 7 test results for shape/vocabulary recognition may be indicative of the general neglect of geometry as a topic in the elementary mathematics program.

TEACHING AND CURRICULUM SUGGESTIONS

2. Utilize activities in which the students have to read, write and identify the name for the shape as well as recognize the shape itself.
3. Performance involving identifying the terms parallel, perpendicular and intersect was definitely poor. It might pay to advertise these terms by displaying very large, colorful posters, e.g., A picture of a skier captioned: Are you a Good Skier? Do Your Skis Run Parallel or Intersect? Or, a picture of a prisoner trying to escape and saying, "I wish I had never known these perpendicular walls or these parallel stripes."

OBJECTIVE RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
38. G–I–B–3 (Grade 7) 52% Identify Different Types of Lines Items 201–205 Given models of lines or line segments, some of which are parallel, intersecting, and perpendicular, the learner will identify and name each. SAMPLE: Which pair of line segments intersect? 	1. These results are vey low. The vocabulary of these geometric concepts is needed not only for subsequent mathematics study but also in everyday life. 2. Only 43% of the students could identify the pair when asked, "Which pair of lines is perpendicular?" Nineteen percent chose the picture showing a pair of parallel lines. 3. The students seemed to do slightly better with "parallel". Sixty-nine percent chose the word parallel to go with the picture of a pair of parallel lines. Unfortunately, the word perpendicular was not a distractor. However, in Item 204, they were required to identify the picture showing parallel lines-73% were correct and only 9% chose the picture of a pair of perpendicular lines. 4. The objective is definitely minimal. The 4-6 program must be adjusted to allow for adequate attention for these concepts.	4. Plane is not a particularly difficult word to read, but the concept needs attention. Discuss airplane, Why is it called a plane? Does it fly through planes in the air, or are its wings the planes that give it the name? Have student hold and stretch out a large piece of cloth and move it to illustrate the many planes in space. (If it is indeed large they won't forget it.) Discuss pictures of various objects and decide if they are suitable representations of planes.
39. G–I–B–5 (Grade 7) 73% Identify Surfaces Representing Plane or Part of Plane Items 206–210 Given the description of a plane, and part of a plane, the learner will identify surfaces which represent a plane or part of a plane. SAMPLE: A plane is a flat surface which extends outward in all direction. For items 206-210, choose the object which suggests a plane. (Sample answers: postage stamp, envelope, notebook paper.)	1. A definition of a plane was given on the test page, but this did not appear to help 27% of the students who did not attain this objective. It is interesting to note that out of the 73% who attained the objective, 68% answered all 5 items correctly, which rules out guesswork and tends to show that these students had explored this objective and knew what was expected of them. Those who answered incorrectly made errors that were equally distributed with no apparent pattern, and this seems to indicate that these students had very little work with the concept of a plane. 2. The objective is minimal.	

ALGEBRA AND GRAPHING

OBJECTIVE / RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
35. AL–1 (Grade 4) 91% Addition or Subtraction Using Placeholder Items 16–20 Given a statement of equality involving addition or subtraction facts and a placeholder for the sum or difference, the learner will supply the sum or difference. SAMPLE: 17 – 8 = □	1. Placeholder notation seems very well learned, with about 90 percent correct on each item. The range was 89-94. 2. While classified as algebraic questions, the items really test facility with the addition and subtraction facts. Most errors occurred by choosing an answer 1 more or 1 less than the correct answer. This suggests difficulty with counting to find an answer rather than difficulty with placeholder ideas.	1. Placeholder and letter notation for variables seem to be well established for pupils. This is true particularly for simple number sentences not involving too many numbers or computations. 2. Performance on items requiring > and < is low enough to raise questions about adequacy of teaching them. It seems that ideas relating to inequalities are known but that children may be having trouble recognizing the correct symbol.
40. AL–2 (Grade 7) 77% Statement of Equality/Supply Missing Number Items 86–90 Given a statement of equality involving addition, subtraction, or multiplication facts and containing a placeholder or letter, the learner will identify the missing number. SAMPLE: n – 3 = 10	1. Three of the five test items involved addition or multiplication. These items were done correctly by 85-90 percent of the students. Two items involving subtraction with a missing sum proved to be more difficult. (n – 3 = 10, 67% correct, and n – 5 = 7, 81% correct.) 2. The objective is minimal. The difficulty appears to be with arithmetic rather than with the algebraic idea of using a letter as a variable.	3. The very poor performance on the number sentence form of the distributive property raises serious doubt about its use in this form to teach multiplication facts or the multiplication algorithm. Beginning work with the distributive property should probably use word names followed with explicit attention to the way the ideas are symbolized. It could be that the heavy symbolization of the distributive property should be delayed until upper grades. grades.
41. AL–4 (Grade 7) 62% Supply Symbol of Equality/Inequality Items 161–165 Given a pair of whole numbers or or number phrases less than 1,000, the learner will identify the appropriate symbol of equality or inequality, < or = or >. SAMPLE: Which symbol should replace the □ to make 7 □ 5 true? (Choices: A. > B. = C. <)	1. Results with these simple sentences are very disappointing. Item results ranged from 65 to 75 percent correct. Twenty-seven percent missed 3 or more items and 60% missed at least one. 2. Incorrect responses on items like 7 □ 5 indicate that the students did recognize < and > as inequality symbols but they did not discriminate which means "is less than" or "is more than". On the items which required some computation, (e.g., 16 + 8 □ 22, 65% correct) incorrect responses were equally distributed between = and the other inequality symbol.	4. The performance on number sentence items throughout both tests suggests that pupils are learning this newer way to express mathematical ideas. When performance was low, it seemed not to be caused by number sentences themselves, but by lack of discrimination of symbols for inequality or symbols for an operation, or by overly complicated sentences.

OBJECTIVE	RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
		3. The format of the questions is not satisfactory because the algebraic number sentence is hidden within the wording of the question. The format alone, however, cannot account for the overall very poor performance on these test items. 4. The objective is minimal.	5. Poor performance on graphing probably reflects lack of emphasis in the curriculum. This skill, very useful in science and social studies, is easy to teach. The results shown that one major component is already learned by pupils – using two numbers to name a point of intersection. The major task is to teach the proper order–horizontal number first and vertical number second. Use activities such as the following: a. Place various small pictures on a large coordinate grid. Students locate each picture by naming pairs of coordinates. b. Class plays Tic-Tac-Toe on the coordinate grid, boys versus girls. Taking alternate turns, a member of a team calls out an ordered pair of numbers and the teacher places the checkers. Four in a row wins. c. Locate towns on a map having coordinate letters and numbers. d. Graph equivalent fractions on the coordinate grid: $\frac{1}{2}$ graphs as (2,1), (4,2), (6,3), (8,4), (10,5) etc.
42. AL–5 (Grade 7) Complete Equation with One/Zero Items 46–50 Given an equation involving one or zero, the learner will complete the sentence. SAMPLE: $6 \times 0 = \square$	85%	1. On each of the five test items, the percent correct ranged from 83 to 93. 2. The most difficult item was the sample, $6 \times 0 = \square$, with almost all those incorrect choosing 6 as the answer. Of almost equal difficulty was the equation $1 \times 11 = \square$ with those incorrect choosing 12 most often. Evidentally, they read x as plus.	
43. AL–7 (Grade 7) Distributive Property/Supply Missing Value Items 66–70 Given a numerical statement involving the distributive property and a placeholder, the learner will insert the missing value. SAMPLE: $4 \times (6 + 7) = (\square \times 6) + (4 \times 7)$	52%	1. Thirty-six percent missed 3 or more items and 67% missed at least one. The percent correct on each item ranged from 58 to 70. 2. The occurance of several numbers and several operation symbols in one sentence seems to make this type of test item too complicated for a great number of students. 3. The writers strongly believe that a working knowledge of the distributive property is essential. Nearly all students, by the end of grade 6, should be able to apply the distributive property to an example like 6×34. We believe, however, that objective AL–7 in this symbolic form is not minimal for grades 4-6.	

ALGEBRA AND GRAPHING (continued from page 44)

OBJECTIVE / RESULTS	REMARKS	TEACHING AND CURRICULUM SUGGESTIONS
44. AL–10 (Grade 7) 72% Identify Related Algebraic Expression Items 156–160 Given a verbal exercise involving a number in a single operation, the learner will write (identify) a related algebraic expression. SAMPLE: Jenny bought 2 candy bars each day for 4 days. How many candy bars did she buy altogether? Choices: 2 x 4 = n 4 ÷ 2 = n 2 + 4 = n 4 – 2 = n.	1. The percent correct on each item ranged from 73 to 81, with the lowest on a division problem and the highest on an addition problem. Twenty-one percent missed three or more. 2. Word problems of this type are certainly minimal for grade 7. The writers feel, however, that there is a reasonable question about whether children should be forced to write number sentences for all problems. 3. The test item does not measure the objective. The objective calls for algebraic expressions, not equations.	
45. P–9 (Grade 7) 22% Locate Points Using a Coordinate System Items 221–225 Given a coordinate system using whole numbers and/or letters and given 3 pairs of coordinates, the learner will locate the points. SAMPLE: What are the coordinates of point M? (M shown on a rectangular coordinate grid. Answer is (6,2)).	1. The most startling result on these poor performance items is that 87 percent missed one or more of the five items. Sixty-four percent missed three or more items. 2. Almost all errors on four items involved the reversal of the coordinates. On the sample item 48 percent chose (2,6) instead of (6,2). On one item where reversed coordinates was not a choice, 80 percent got it correct. Thus, it appears that children can read numbers along two axes and see that a number pair goes with a point of intersection. 3. The writers feel that it is inacceptable to delete coordinate graphing as a major objective in grades 4-6. We urge that the objective be restated so as to be more identifiable with the road map type of coordinate where a letter and a number are used and order is not crucial.	

ERIC

IV

CONCLUDING REMARKS

The test results show some major areas of strength in childrens' knowledge at the end of Grades 3 and 6. These major areas of strength are in pre-number concepts (classification), addition and multiplication concepts for whole numbers, addition and multiplication computation for whole numbers, and algebraic ideas such as placeholders in number sentences.

Major areas of weakness are numeration for whole numbers and decimals, subtraction and division of whole numbers, measurement, fraction concepts, computation with fractions and decimals, and geometry.

Throughout the content areas are weaknesses with vocabulary, interpreting diagrams, and ordering objects and numbers.

Each area of perceived weakness is discussed briefly and suggestions by the writers on possible causes and possible remedies are given. The reader is warned that the following are interpretive comments by the writers.

1. TECHNICAL VOCABULARY AND SYMBOLS

Every subject area has technical vocabulary which must be learned in order to communicate ideas to other people. Often this vocabulary is also essential in learning new concepts. While it is easy to build a list of mathematical words which is excessive even for the most able students, the vocabulary used in this test does seem minimal.

Analysis of test results shows that the following simple words, phrases, and symbols cause trouble for many students:

Same size	AM
Same shape	PM
Order from full to empty	>
Fewer and fewest	<
Just before	Shaded (in connection with fraction diagrams)
Just after	Parallel and perpendicular
Even numbers	Intersecting
Twice as many	Rectangle
Congruent	Plane

There were no adequate test items on other essential mathematics vocabulary such as addend, sum, factor, product, and quotient. Other test data suggest that these words are not familiar to many students. Eliminating words which are only moderately necessary and giving more stress to the essential words is a positive direction for the curriculum.

Vocabulary must be taught. Vocabulary is not learned incidentially. It needs direct attention in classrooms, frequent maintenance, and frequent usage by both teachers and pupils. Vocabulary should not be taught in an isolated fashion, but with the conepts for which the words are names.

Errors occurred often with words and symbols which are antonyms. Pupils made reversal errors on opposites such as "Before-After", "Greatest-Least", "< and >", "AM and PM", and "Full and Empty". When opposites are being taught, it is probably best to teach one of the words until it is learned well by most pupils. Then teach the opposite. For example, teach "Greatest" and get it established well

before teaching "Least". Similarly, teach ">" well before introducing "<".

Mathematical symbolism, contrary to English language expression, is not redundant. Each symbol must be perceived, its meaning recognized and thought about. There is substantial evidence that many children do not discriminate between symbols, nor do they read them carefully. For example, the symbol "×" was often interpreted as "+" and conversely. Such discrimination and recognition is usually so obvious to adults that we often forget that we must help children learn to do it. Mathematical symbols must be read carefully.

2. MAKING AND USING DIAGRAMS

Difficulties were apparent in interpreting diagrams for subtraction, for fractions, for decimals, for area, and for volume. Since interpreting and drawing diagrams are essential skills in other subjects as well as mathematics, special attention should be given to them.

Diagrams usually result from work with concrete objects. Diagrams become a way to represent thought in a compact, efficient way; for example, a child may use a strip of cardboard with 10 blocks on it for ten. With only a little help, the child can draw a picture of the strip such as $\boxed{}$ to represent ten and $\boxed{}$ to show just one. The diagram drawn by the child helps him represent his thought in a compact and visual way. Similarly, a child can build ideas folding pieces of paper to show fractions and then draw diagrams to show the piece of paper and the parts of that paper.

The use of diagrams at all levels is to be encouraged. Recognition and interpreting diagrams should be accompanied by explicit help in reproducing and drawing diagrams.

3. ORDER AND SEQUENCE

It is difficult to know if trouble with order and sequence is caused by erroneous concepts or by inadequate vocabulary. Probably both are somewhat responsible. It is evident, whatever the cause, that pupils do not do well on ordering tasks.

Ordering of two-digit and three-digit numbers may reflect inadequate developmental work on base and place value. Other difficulties are caused by words such as "just before", "just after", and "between". Concrete objects or diagrams for tens and hundreds arranged "in order" should be related to the needed vocabulary using a variety of words.

4. NUMERATION

The most important topic in elementary school mathematics is numeration. Much of the daily use of mathematics involves understanding of numbers expressed in place value notation. All algorithms for whole numbers have numeration as a prerequisite skill.

In view of the importance of numeration, performance on the test items clearly is not acceptable. Teaching suggestions were given on pages 9-15. Other courses of action such as the following should be considered:

a. Allot more time in the curriculum at each level to teaching numeration.

b. Make greater use of physical objects in making tens and hundreds.

c. Teach students to draw diagrams for tens, hundreds, and thousands.

d. Place greater attention on relating objects or diagrams to the word names and symbols such as "eighty-one" and "81".

e. Begin concrete objects and diagrams for "tenths" and "hundredths" earlier, perhaps ages 8 or 9, for subsequent use with money and the metric system.

f. Teach the word names, e.g. 3 tens 7 ones as well as expanded notation 30 + 7.

In view of the importance of numeration, the writers feel that the topic was not adequately tested. Major ideas were not assessed. Consideration should be given also to revision of the objectives themselves to place more stress on drawing and using diagrams for numeration ideas.

5. SUBTRACTION OF WHOLE NUMBERS

The performance on subtraction items suggests the need for a better analysis of the algorithm, more attention to the concept of subtraction, and more stress of the application of subtraction to "How many more?" situations. It seems clear to the writers that subtraction without renaming should be mastered by most pupils before proceeding to subtraction with renaming. Of major importance is a more careful analysis of all the steps in the subtraction algorithm. (See pages 18 and 19 for suggestions.)

6. DIVISION OF WHOLE NUMBERS

Suggestions are given on pages 23-24 for improving division. The beginning algorithm with 1-digit division should be mastered before moving on to more complicated problems. Furthermore, care is needed in analyzing and teaching each of the main steps in division.

The writers raise a serious question about stressing repeated subtraction in doing division. The writer think that

ERIC

greater emphasis on relating multiplication and division may be more productive.

7. MEANING OF FRACTIONS

While not tested as directly as possible, it seems clear that there is a major inadequacy in quantitative thinking with fractions. Many pupils are operating on the symbols rather than thinking about a quantity or an amount of something. Errors such as $\frac{2}{3} + \frac{1}{3} = \frac{3}{6}$ show such symbolic work without the quantitative ideas.

The major recommendation of the writers is that substantially more time, emphasis and experience be provided for the quantitative ideas related to fractions. By quantitative ideas is meant ideas of how much, what part of, etc., always stressing that the answer be given in relation to the unit chosen. This would mean more time in using objects such as sheets of paper as units, more time in making and interpreting diagrams, and firmly relating fraction ideas to fraction symbols <u>prior</u> to algorithmic work. Little improvement can be expected on the algorithms until the initial idea of fractions is understood well by pupils.

8. COMPUTATION WITH FRACTIONS

Test items on computation with fractions did not reflect content usually found in the curriculum such as addition and subtraction with unlike fractions, multiplication with fractions (test used only unit fractions) or division with fractions. The content on operations in the test is minimal, indeed. Yet performance was low. Improvement can probably

only be made by first improving the initial work with the meaning of fractions. Then there must be very careful development of the algorithms, making sure they are related meaningfully and firmly to the initial ideas.

Improvement in computation will not come simply by providing more practice. Time would be better spent thinking about and working on improving the development of the algorithm in a meaningful way. Extensive use of concrete objects and diagrams to stress quantitative thinking seem essential.

9. MEANING OF DECIMALS

The meaning of decimals was not assessed adequately because of the distractors for the items and the test questions themselves. Poor performance on computation items such as .3 + .24 suggests strongly that the meaning of decimals is not even close to minimal expectations.

The meaning of decimals should be developed from numeration ideas — moving to the right of the ones place — and from fractions with denominators of 10 or 100 or 1000. The use of concrete materials such as a 100 square can help build the quantitative notions needed for decimals.

The meaning, symbols, and word names for decimals probably should be introduced earlier. With the change to the metric system, decimals will be needed earlier. It is feasible to introduce "tenths" at age 8 or 9 with "hundredths" presented after mastery of "tenths".

ERIC

10. OPERATIONS WITH DECIMALS

Computation with decimals must be firmly rooted in the understanding of their meaning. Done properly, the operations can reinforce place value ideas for whole numbers. Conversely, understanding of place value for whole numbers and of the algorithm for whole numbers can help in learning operations with decimals.

It should be noted that multiplication and division with decimals — usually taught before grade 7 — were not tested.

11. MEASUREMENT

Next to number itself, measurement is probably the most useful part of elementary school mathematics. Test results indicate that this vital topic is a victim of poor instruction and that not enough time is devoted to it.

In linear, area, and volume measurement the fundamental and pervasive idea of unit is not learned well. Initial work using non-standard units such as "toothpicks", "straws", "cards", or "blocks" should help build the idea of unit.

From an understanding of repeating "length units" such as "toothpicks" or "straws", a number line or ruler scale can be developed (see suggestions on pages 9 and 35). This understanding can help improve poor performance with both rulers and number lines. Deeper understanding of the way scales and number lines are made seems essential to any major improvement in measurement skills.

Explicit attention is needed on reading scales of various kinds — rulers, weighing scales, thermometers — with

ERIC

special instruction on what the various marks on scales represent. Instruction on scales and their markings should be accompanied by extensive experience handling and using a variety of measuring instruments.

12. GEOMETRY

The overall poor performance on geometry items reflects the lack of attention the ideas receive in the curriculum and inadequate focus on the relatively minimum vocabulary needed.

Geometric ideas and vocabulary are needed in everyday affairs. Often they are easier than those related to number and computation. And many times geometry can help develop number ideas. The writers are of the strong opinion that time must be provided for geometry at each level of school.

Graphical work showed low performance but could be improved substantially just by teaching the order of numbers in a pair for locating a given point. Graphing is so useful in other subject areas, especially science and social studies, that its inclusion in the curriculum should be assured.

* * * * *

There are usually many reasons for poor performance on the mathematics tests -- instructional materials, home background, motivation, etc. Teachers, together with administrators and other curriculum leaders, need to discuss what needs emphasis in a local school. There should be a commitment to establish priorities for the mathematics program. This requires that the local staff have time to sit together to analyze and interpret their local results.

Individual teachers will want to assess the emphasis given certain topics at the level he or she teaches. Diagnosis of strengths and weaknesses of his or her pupils should become an integral part of lesson planning and curriculum building. The total staff of an elementary school may wish to meet and communicate what each level contributes to the development of certain skills and concepts.

Some of the observed weaknesses in performance are the result of inadequate instructional materials. Often these materials do not devote enough space to a topic; nor do they provide enough developmental work. Next to be examined as a possible cause for weakness is the extent concrete objects and diagrams are used to develop and to promote quantitative thinking. There appears to be too much stress on the purely symbolic part of the topic when it is introduced.

The kind and amount of attention given to language development in the mathematics program needs examining. We have already mentioned the need for teaching the mathematics vocabulary. Unlike some other subjects, students do not have many opportunities to learn the required vocabulary in other situations.

Whatever the separate or combined reasons for poor test performance, the most critical inadequacy under the control of the school is in the initial developmental work for the major ideas. Overall improvement will be most obvious when initial work is done in a better way. A sound beginning seems to the writers to be the most valuable assist any pupil can have. It is <u>the</u> thing most likely to improve the attainment of valuable learning objectives.

APPENDIX A

STATEWIDE RESULTS

SUMMARY

ERIC

MICHIGAN EDUCATIONAL ASSESSMENT PROGRAM 1973-74 (YEAR 5)

SUMMARY FOR
MICHIGAN STATEWIDE

CLASSROOM, SCHOOL AND DISTRICT SUMMARY FORM

DISTRICT CODE -

SCHOOL CODE

GRADE FOUR

DATE TESTED 11/73

WORD RELATIONSHIPS

STAND. SCORE INTERVAL	FREQUENCY	PERCENT & ABOVE
70-74	1810	1
65-69	9089	6
60-64	13675	[illegible]
55-59	16112	10
50-54	19353	12
45-49	29028	18
40-44	24735	16
35-39	21728	14
30-34	14756	[illegible]
25-29	6270	4
20-24	1733	1
15-19	157	1
158446	NUMBER OF PUPILS	

PERCENT OF STUDENTS ACHIEVING						%	NUMBER OF PUPILS	OBJ. NO.	OBJECTIVE	OBJECTIVE CODE
									****** MATHEMATICS ******	
3	5	4	2	6	79	85	158073	1	INDICATE OBJECTS THAT ARE SAME SIZE	AR-I-A-6
3	3	2	2	4	84	90	158144	2	INDICATE SIMILAR GEOMETRIC SHAPES	[illegible]
9	10	6	4	8	63	71	157721	3	INDICATE OBJECTS ARRANGED FULL TO EMPTY	AR-I-A-16
3	2	3	3	4	84	90	157957	4	INDICATE LONGEST AND SHORTEST OBJECTS	[illegible]
3	3	4	2	3	[illegible]	87	158004	5	INDICATE FIRST AND LAST	AR-I-A-40
3	3	4	3	9	7[illegible]	84	157398	6	CHOOSE EQUIVALENT SETS	[illegible]
17	9	4	3	10	57	67	158211	7	CHOOSE SETS HAVING FEWER NUMBERS	AR-I-B-32
[illegible]	9	7	11	15	51	6[illegible]	15[illegible]	8	INDICATE APPROPRIATE NUMERAL FOR POINT ON A LINE	[illegible]
1	3	6	4	9	77	86	158775	9	CHOOSE GREATEST AND LEAST NUMBER	AR-I-B-43
3	4	3	2	4	8[illegible]	8[illegible]	158481	10	CHOOSE NUMBER BETWEEN TWO NUMBERS	[illegible]
2	4	5	9	12	68	80	158667	11	CHOOSE NUMBER BEFORE OR AFTER A NUMBER	AR-I-B-45
3	4	4	7	18	6[illegible]	78	157537	12	IDENTIFY A NUMERAL LESS THAN A 100	[illegible]
3	5	9	12	17	54	71	158530	13	INDICATE NUMBER BEFORE OR AFTER NUMBER WITHIN A DECADE	AR-I-B-65
3	3	3	6	14	73	89	158756	14	INDICATE WHICH OF 2 NUMBERS IS GREATER OR LESS	[illegible]
3	6	6	8	14	64	78	155641	15	INDICATE THE VALUES OF A SET OF DIMES AND PENNIES	AR-I-B-70
2	5	7	12	40	3[illegible]	74	158290	16	IDENTIFY CORRECT EXPANDED NOTATION	[illegible]
8	10	9	9	19	45	64	158592	17	CHOOSE LIST OF NUMBERS IN ASCENDING ORDER	AR-I-B-81
[illegible]	2	4	7	16	7[illegible]	86	158719	18	INDICATE GREATER OR LESS/ SCRAMBLED POSITIONS	[illegible]

PERCENT OF STUDENTS ACHIEVING						%	NUMBER OF PUPILS	OBJ. NO.	OBJECTIVE	OBJECTIVE CODE
2	6	6	10	22	55	77	158688	19	INDICATE NEXT NUMBER IN A SEQUENCE	AR-I-B-84
[illegible]	11	8	6	8	6[illegible]	68	158601	20	INDICATE A NUMBER THAT IS A MULTIPLE OF 2	[illegible]
4	16	23	10	16	31	47	157053	21	SELECT SET WITH TWICE AS MANY MEMBERS AS ANOTHER	AR-I-B-86
[illegible]	[illegible]	[illegible]	[illegible]	12	[illegible]	[illegible]	158843	22	ADD TWO-DIGIT AND ONE-DIGIT NUMBER/NO CARRYING	[illegible]
1	2	3	7	30	56	86	158355	23	IDENTIFY OPERATION AND SUM OR DIFFERENCE / NO BORROWING	AR-II-B-6
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3[illegible]	[illegible]	[illegible]	24	NUMBER SENTENCES / SUBTRACTION	[illegible]
1	2	3	8	24	61	85	158266	25	NUMBER SENTENCES / ADDITION OR SUBTRACTION-IDENTIFY OPERATION	AR-II-B-11
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	26	NUMERICAL SET COMPARISONS	[illegible]
1	3	4	5	12	75	87	158826	27	SUBTRACT ONE-DIGIT FROM TWO-DIGIT NUMBER / NO BORROWING	AR-II-B-15
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6[illegible]	[illegible]	[illegible]	28	SUBTRACT TWO-DIGIT FROM TWO-DIGIT NUMBER / NO BORROWING	[illegible]
8	21	29	28	10	5	15	156326	29	IDENTIFY FRACTIONAL PART OF SET OF OBJECTS	AR-III-D-1
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	30	MEASURE OBJECTS TO NEAREST INCH	[illegible]
1	3	3	4	14	75	89	157853	31	TELLING TIME	M-II-A-6
[illegible]	[illegible]	[illegible]	11	[illegible]	[illegible]	[illegible]	[illegible]	32	IDENTIFY GREATEST OR LEAST AMOUNTS OF MONEY	[illegible]
3	6	8	13	49	22	71	155854	33	IDENTIFY TEMPERATURES	M-II-C-1
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	34	IDENTIFY GEOMETRIC SHAPES	[illegible]
1	1	2	5	14	77	91	158851	35	ADDITION OR SUBTRACTION USING PLACEHOLDER	AL-1
									******** READING ********	
[illegible]	30	[illegible]	13	20	4[illegible]	6[illegible]	[illegible]	1	MATCH WORDS WITH DEFINITIONS	2.1
9	9	8	7	12	54	66	157254	2	INDICATE PHRASES WITH SAME MEANINGS	2.2
[illegible]	11	21	22	28	11	[illegible]	[illegible]	3	ANSWER QUESTIONS WITH WORDS/PHRASES FROM SELECTIONS	3.1
8	12	9	9	12	50	62	156011	4	CHOOSE WORD APPROPRIATE TO BLANK SPACES (CLOZE PROCEDURE)	3.2
[illegible]	4	5	7	18	6[illegible]	[illegible]	[illegible]	5	IDENTIFY METHOD OF ARRANGING DATA	4.1
5	11	14	16	20	34	54	157030	6	ALPHABETIZE WORDS THROUGH FIRST 3 LETTERS	4.4
[illegible]	8	7	6	33	4[illegible]	7[illegible]	[illegible]	7	INDICATE FACTUAL SELECTIONS	5.1
7	10	8	4	8	63	71	157516	8	INDICATE FICTIONAL SELECTIONS	5.2
[illegible]	10	12	12	1[illegible]	4[illegible]	61	156453	9	INDICATE AUTHOR'S PURPOSE	6.1-6.3
4	11	15	18	25	26	51	165468	10	INDICATE TITLE MOST APPROPRIATE FOR SELECTION	7.1
[illegible]	10	14	22	28	2[illegible]	50	157064	11	INDICATE PICTURES BEST DESCRIBING MAIN IDEA IN SELECTIONS	7.2
8	18	20	19	19	17	36	155389	12	CHOOSE BEST SUMMARY OF A SELECTION	7.3
[illegible]	[illegible]	18	19	23	2[illegible]	45	156543	13	MATCH QUOTATION FROM STORY WITH SPEAKER	8.4
8	17	18	19	21	17	38	156447	14	ANSWER QUESTIONS RELATING TO SEQUENCE IN STORIES	9.3
[illegible]	20	23	20	17	1[illegible]	28	157191	15	CHOOSE SELECTIONS ALIKE IN MOOD	10.2
4	10	12	12	19	43	62	156376	16	CHOOSE SENTENCE BEST DESCRIBING HOW CHARACTER FEELS IN STORY	10.3
6	14	14	13	22	3[illegible]	54	155676	17	CHOOSE PHRASE BEST DESCRIBING WORK IN FIGURATIVE LANGUAGE	10.6
5	11	12	12	17	43	60	155353	18	MATCH CAUSES WITH EFFECTS	11.1
[illegible]	13	17	18	22	2[illegible]	48	156217	19	CHOOSE MOST APPROPRIATE CONCLUSION FOR A STORY	11.2
4	9	14	21	26	26	52	158234	20	ANSWER LOCATIONAL QUESTION ABOUT REFERENCE SOURCES	13.1
[illegible]	8	9	18	29	3[illegible]	67	158164	21	ANSWER LOCATIONAL QUESTION ABOUT NEWSPAPERS	13.2
5	11	16	19	24	26	50	156500	22	INDICATE MEANINGS, GENERALIZATIONS, CONCLUSIONS NOT EXPRESSED	14.1-14.3
[illegible]	11	12	12	18	4[illegible]	59	156417	23	ANSWER QUESTIONS RELATING TO MOTIVATION	

MICHIGAN EDUCATIONAL ASSESSMENT PROGRAM 1973-74 (YEAR 5)

SUMMARY FOR MICHIGAN STATEWIDE

CLASSROOM, SCHOOL AND DISTRICT SUMMARY FORM

DISTRICT CODE

SCHOOL CODE

GRADE SEVEN

DATE TESTED 11/73

WORD RELATIONSHIPS		
65-69	12631	8
55-59	30595	18
45-49	24575	15
35-39	17191	10
25-29	2611	2
15-19	103	1
167391	NUMBER OF PUPILS	

PERCENT OF STUDENTS ANSWERING							NUMBER OF PUPILS		OBJECTIVE	OBJECTIVE CODE
3	5	6	11	23	53	76	168178	1	IDENTIFY NUMBER 100/1000 LARGER	AR-I-B-8T
0	1	1	3	14	80	94	168170	3	ADD 3- AND 1-, 2-, AND 3- DIGIT NUMBERS	AR-II-A-22
4	5	5	8	21	57	78	168124	5	SUBTRACT 2- OR 3- DIGIT NUMBER FROM 3- DIGIT NUMBER	AR-II-B-20
1	2	2	2	8	85	93	167999	7	INVERSE MULTIRLICATION	AR-II-C-11
2	5	5	6	17	65	82	167819	9	MULTIPLY 2-DIGIT AND 1-DIGIT NUMBERS	AR-II-C-15
5	7	5	3	5	75	80	167677	11	REWRITE DIVISION FACT AS MULTIPLICATION FACT	AR-II-D-7
7	12	10	9	13	49	62	167525	13	DIVISION / ONE-DIGIT DIVISOR, FOUR DIGIT DIVIDEND	AR-II-D-15
6	7	6	5	13	63	76	167071	15	IDENTIFY SHADED AREA OF FIGURE WITH FRACTION	AR-III-A-18
15	9	7	5	9	54	65	166953	17	IDENTIFY SUM OF 2 FRACTIONS WITH LIKE DENOMINATORS	AR-III-B-4
5	10	10	12	16	47	63	166811	19	SUBTRACT FRACTION FROM MIXED NUMBER / LIKE DENOMINATORS	AR-III-C-4
8	10	7	5	10	60	70	166696	21	MULTIPLY 2 UNIT FRACTIONS	AR-III-D-4

									OBJECTIVE	OBJECTIVE CODE
40	17	10	7	11	15	26	167191	23	NAME PLACE VALUES OF DECIMAL FRACTION	AR-IV-A-7
7	21	34	24	10		14	166489	25	ADDITION / SUBTRACTION DECIMAL PROBLEMS: TENTHS AND 100THS	AR-IV-B-6
3	10	17	32	20	18	38	165808	27	IDENTIFY PAIR OF SETS / EQUIVALENT RATIO	AR-VI-15
10	17	11	13	23	26	49	164866	29	ESTIMATE AREA OF A POLYGON	M-I-B-8
2	6	15	18	22	37	59	165664	31	TELL TIME	M-II-A-10
2	4	7	12	26	49	75	168098	33	ADD / SUBTRACT: MONEY	M-II-B-7
7	14	15	15	20	29	49	164326	35	SOLVE PROBLEM WITH MONEY VALUES	M-II-B-9
3	8	9	13	18	48	66	165425	37	NAME QUADRILATERALS	G-I-A-4
6	8	8	5	5	68	73	165103	39	IDENTIFY SURFACES REPRESENTING PLANE OR PART OF PLANE	G-I-B-5
5	12	11	10	21	41	62	166098	41	SUPPLY SYMBOL OF EQUALITY / INEQUALITY	AL-5
9	14	13	13	19	33	52	167728	43	DISTRIBUTIVE PROPERTY / SUPPLY MISSING VALUE	AL-2
5	34	25	14	9	13	22	164212	45	LOCATE POINTS USING A COORDINATE SYSTEM	P-9
9	12	11	12	22	33	55	164776	1	[illegible] WORDS WITH DEFINITIONS	18.1
3	7	6	4	12	68	80	165940	3	CHOOSE WORD APPROPRIATE TO BLANK SPACES (CLOZE PROCEDURE)	19.1
4	7	7	10	20	53	73	167229	5	IDENTIFY METHOD OF ARRANGING DATA	20.1
3	7	7	6	11	66	77	167183	7	INDICATE FACTUAL SELECTIONS	22.1
4	9	12	14	22	38	60	166226	9	INDICATE AUTHOR'S PURPOSE	23.1-23.3
2	6	8	13	29	42	71	165695	11	INDICATE PICTURES BEST DESCRIBING MAIN IDEA IN SELECTIONS	24.2
6	12	12	16	26	28	54	165400	13	CHOOSE SELECTIONS ALIKE IN IDEAS EXPRESSED	24.4
4	11	15	17	23	31	54	165813	15	ANSWER QUESTIONS RELATING TO SEQUENCE IN STORIES	26.3
3	8	11	14	20	44	64	165244	17	ANSWER QUESTIONS RELATING TO MOTIVATION	28.1
4	9	12	13	21	41	62	164963	19	INDICATE MEANINGS, GENERALIZATIONS, CONCLUSIONS NOT EXPRESSED	29.5
2	7	8	10	23	50	73	167046	21	ANSWER LOCATIONAL QUESTION ABOUT REFERENCE SOURCES	31.1
5	10	10	13	29	34	63	164553	23	CHOOSE MOST APPROPRIATE CONCLUSION FOR A STORY	32.1

The following were members of the Guidelines Committee for Quality Mathematics Teaching during the development of this monograph:

Maja S. Barr	Saginaw
Richard Debelak	Iron Mountain
Theresa Denman	Detroit
Jacqueline Dombroski	Petoskey
Herbert Harmon	Kalamazoo
Lou Henkel	Grand Rapids
Jean Houghton	Weidman
Daniel Korman	West Branch
Evelyn Kozar	Detroit
Bea Munro	Ann Arbor
Charles Schloff	Dearborn Heights
Geri Westover	Bay City
James K. Bidwell (Chairman)	Mt. Pleasant

Your comments and criticisms of this monograph as well as suggestions for other monographs (or manuscripts for them) can be sent to:

Albert Schulte
Oakland Schools
2100 Pontiac Lake Road
Pontiac, MI 48054